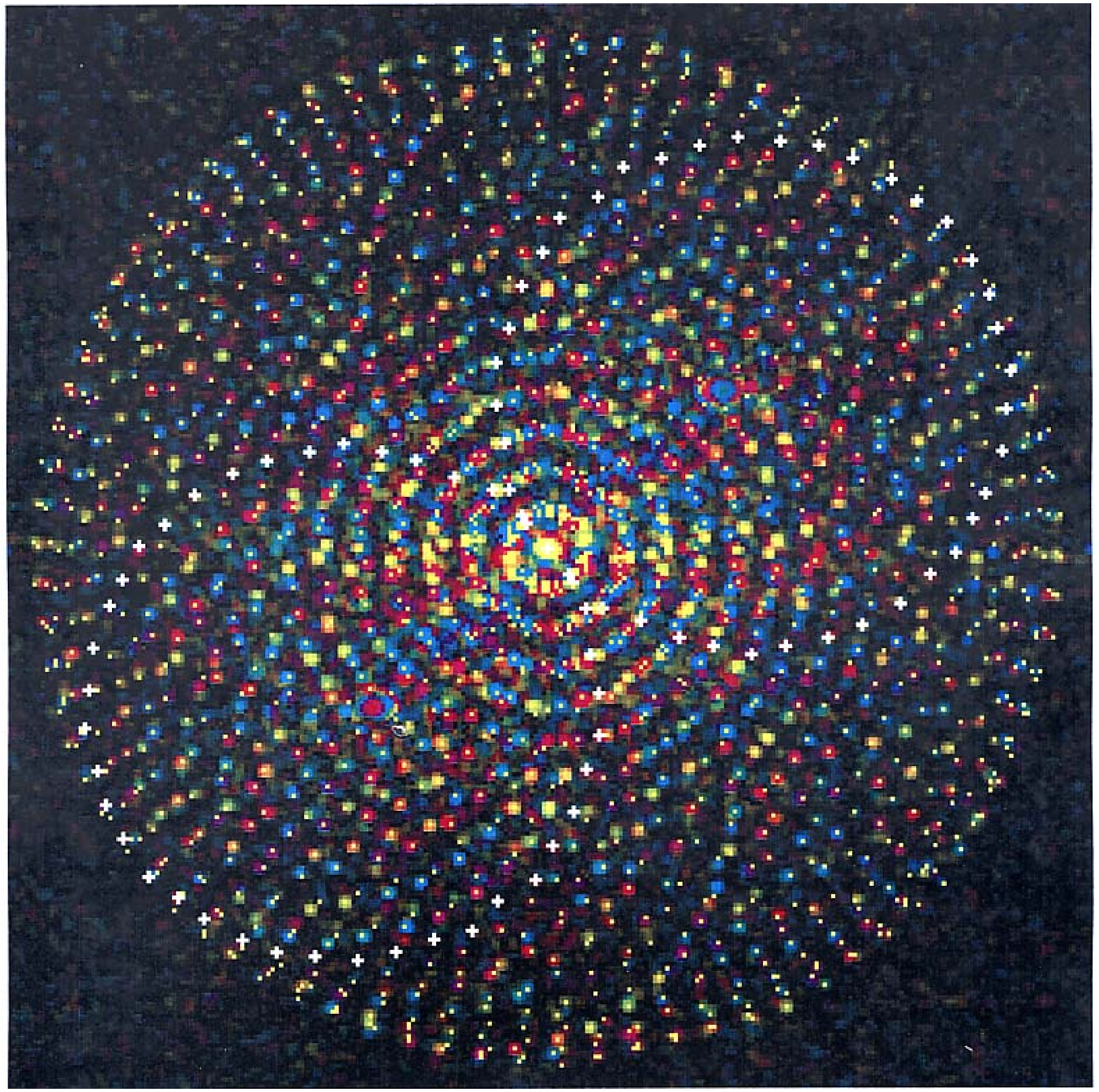
COMPUTATIONAL IMAGING



Berthold K.P. Horn

What is Computational Imaging?

- Computation inherent in image formation

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(1) Computing is getting faster and cheaper

—precision physical apparatus is not

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(2) Can't refract or reflect some radiation

What is Computational Imaging?

- Computation inherent in image formation

(1) Computing is getting faster and cheaper

—precision physical apparatus is not

(2) Can't refract or reflect some radiation

(3) Detection is at times inherently coded

Computational Imaging System

Examples of Computational Imaging:

(1) Synthetic Aperture Imaging

(2) Coded Aperture Imaging

(3) Diaphanography—Diffuse Tomography

(4) Exact Cone Beam Reconstruction

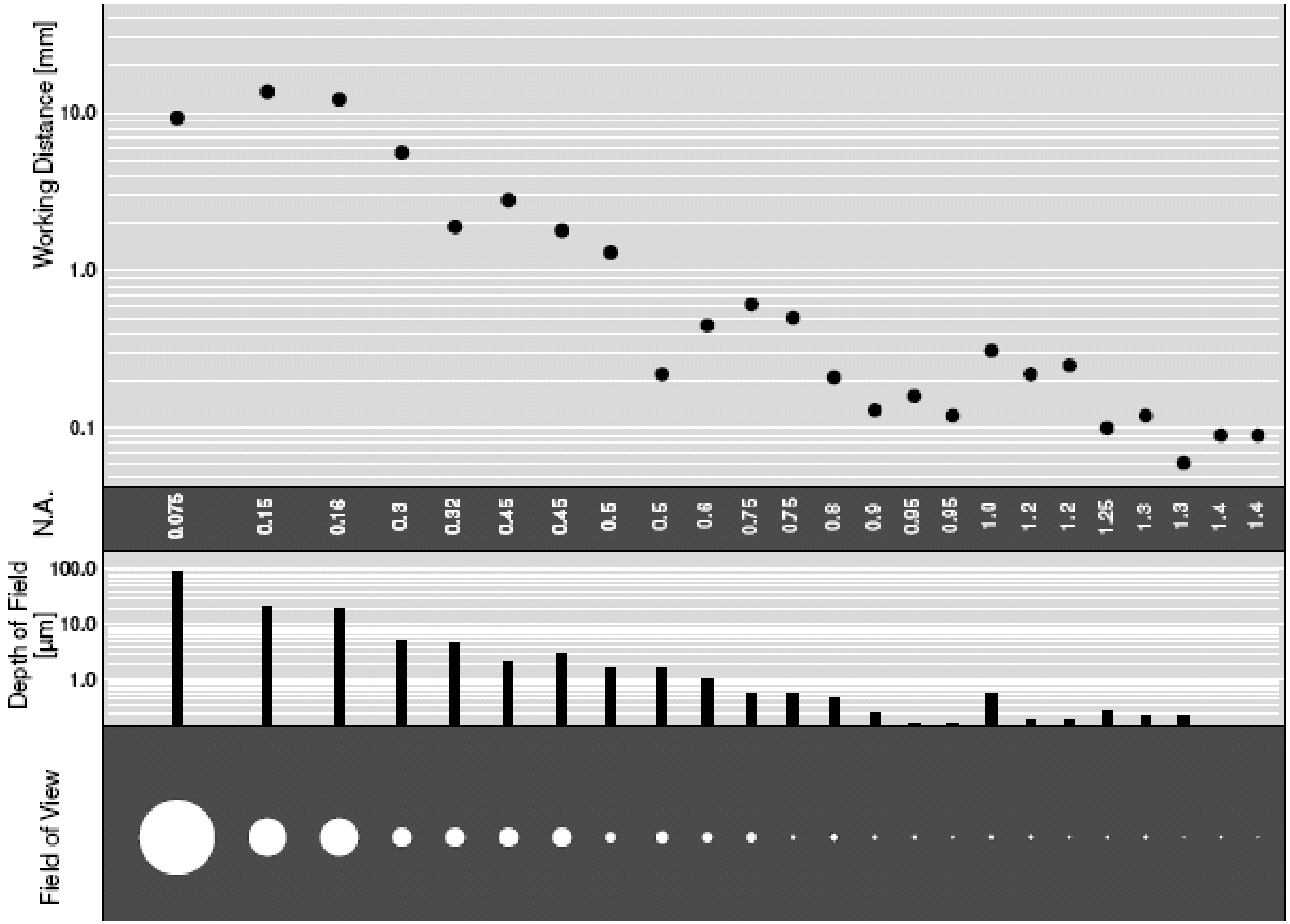
(1) SYNTHETIC APERTURE IMAGING

Traditional approach:

- Coupling of resolution, DOF, FOV to NA
- Precision imaging — “flat” illumination

with: Michael Mermelstein, Jekwan Ryu,
Stanley Hong, and Dennis Freeman

Objective Lens Parameter Coupling



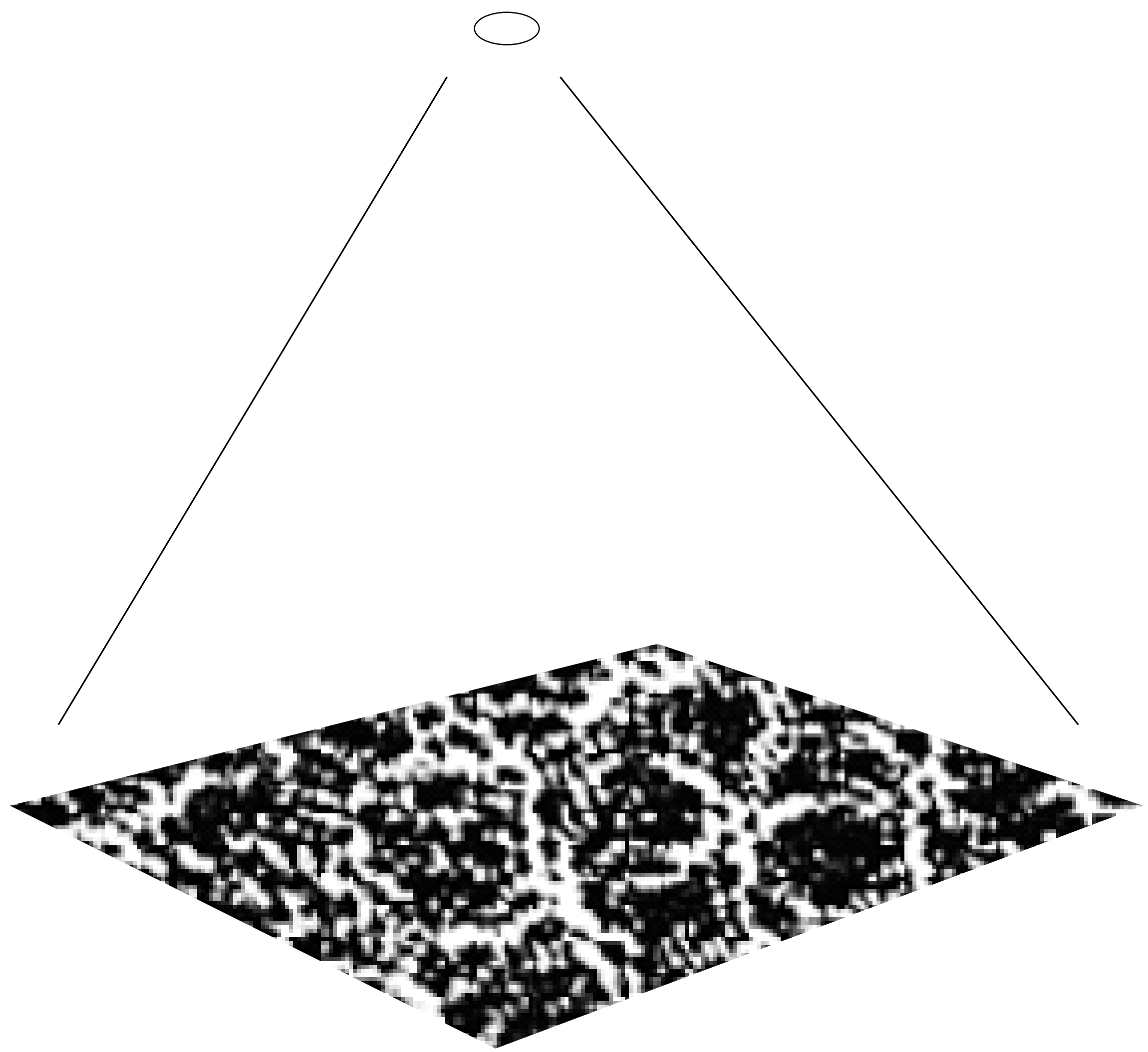
Synthetic Aperture Imaging

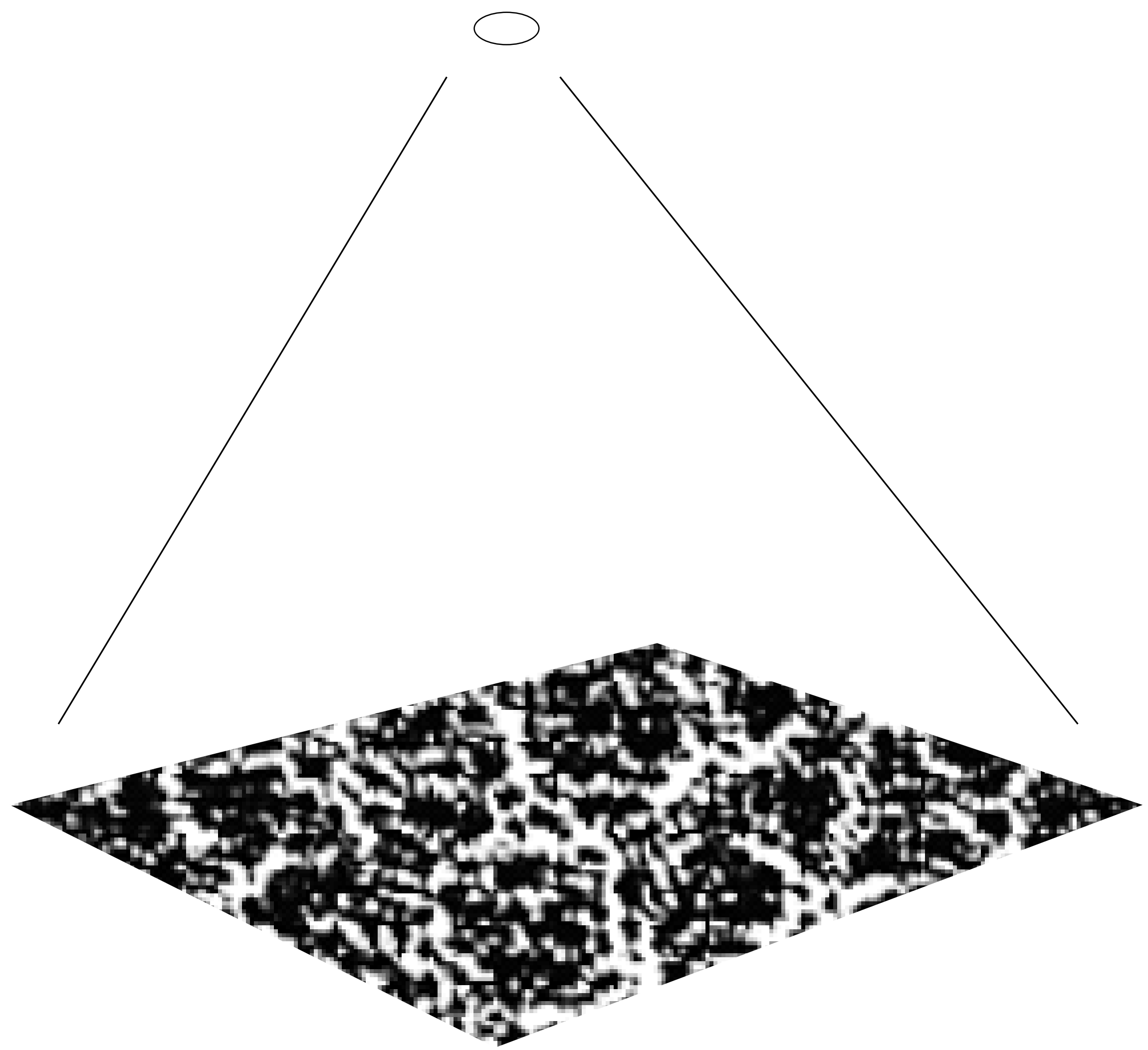
Traditional approach:

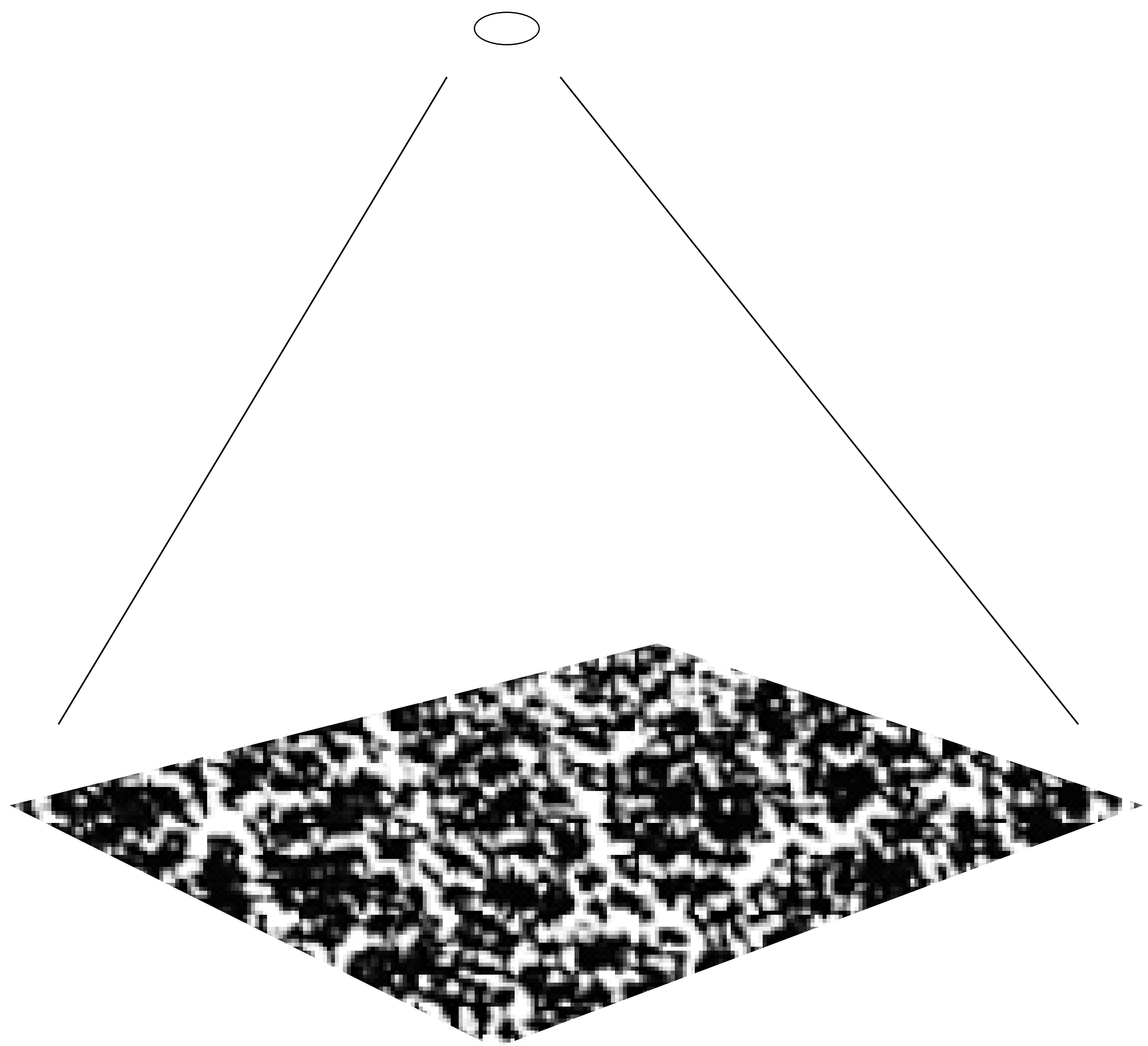
- Coupling of resolution, DOF, FOV to NA
- Precision imaging — “flat” illumination

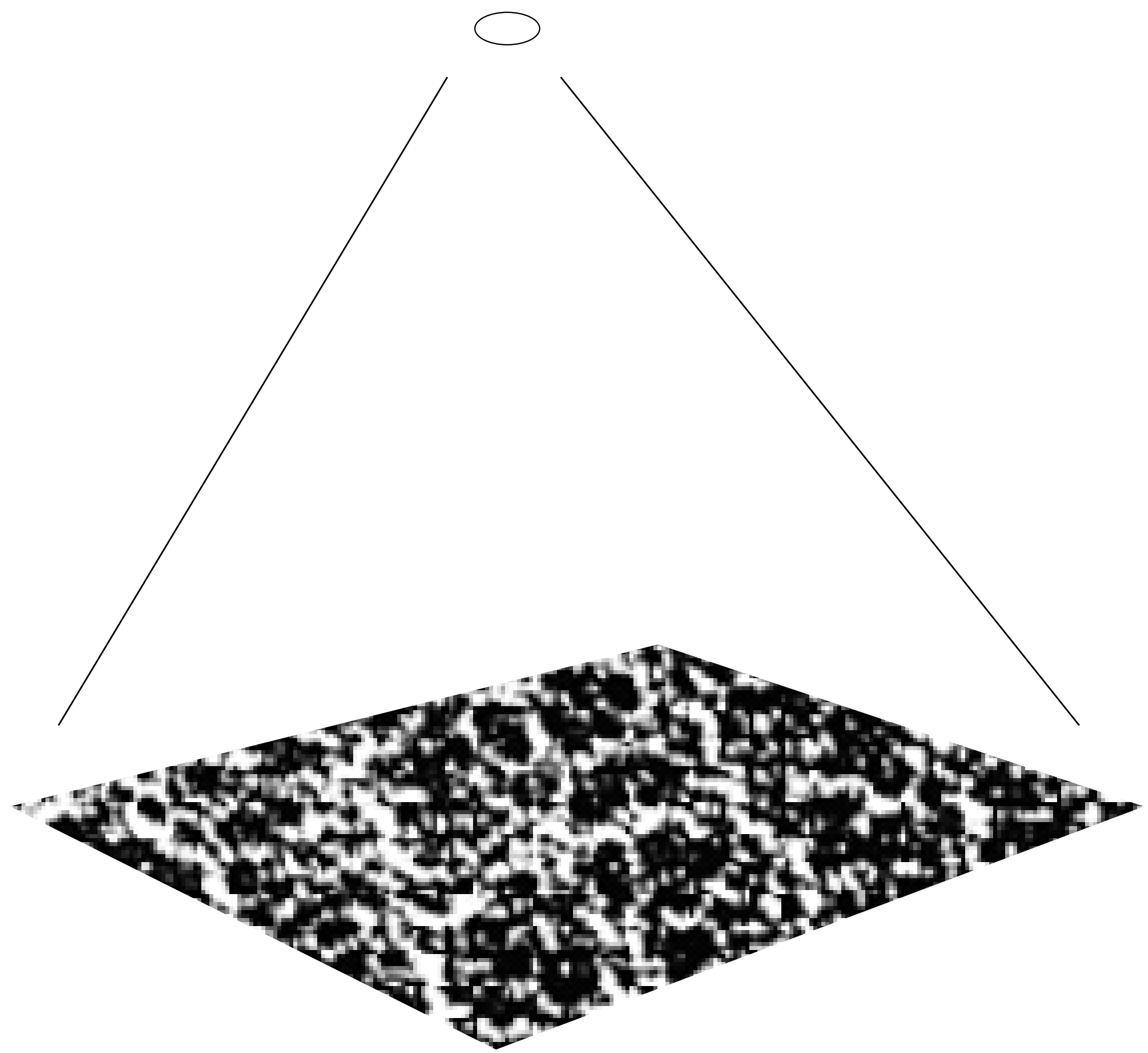
New approach:

- Precision illumination — Simple imaging
- Multiple images — Textured illumination







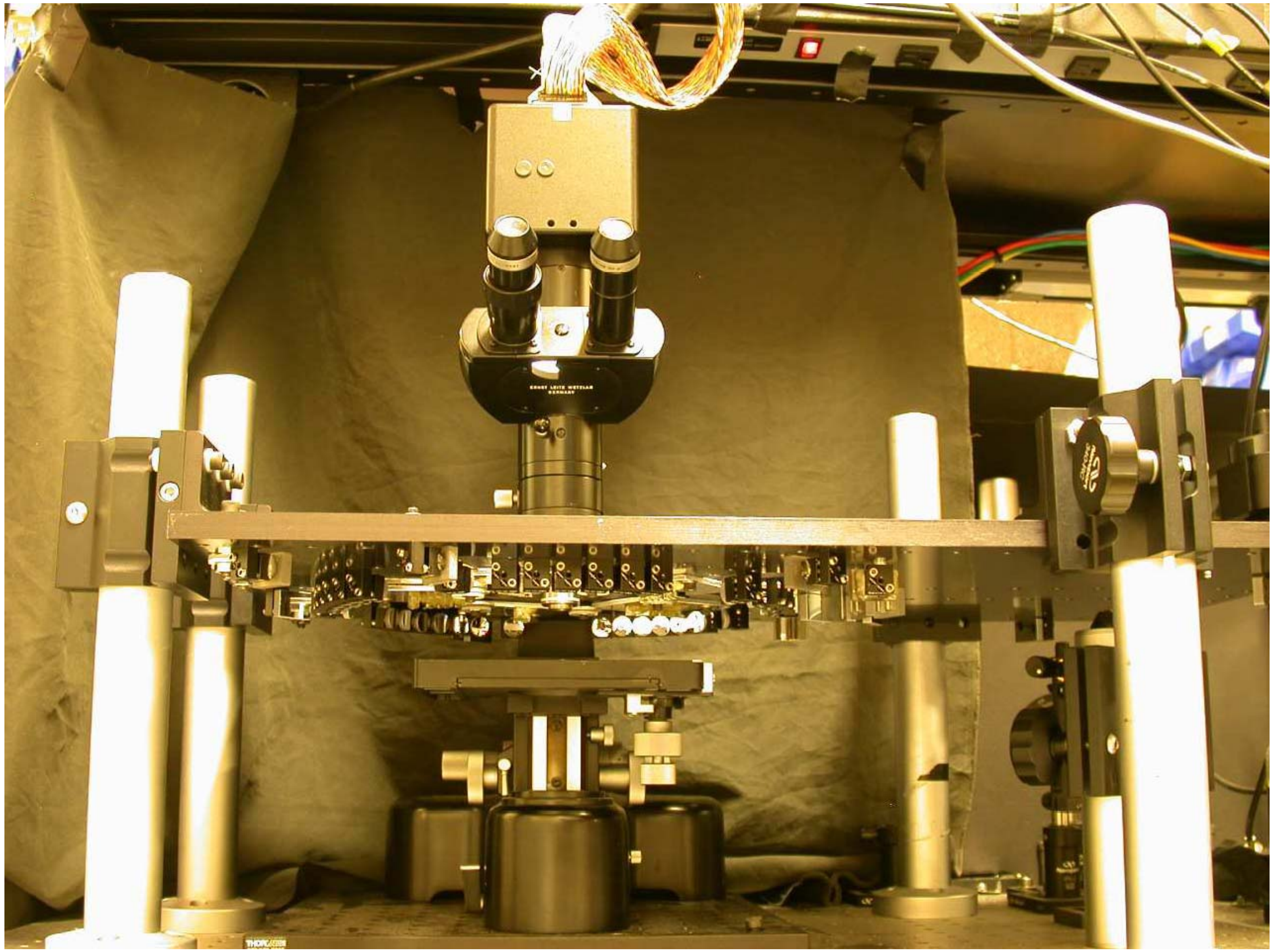


Synthetic Aperture Imaging

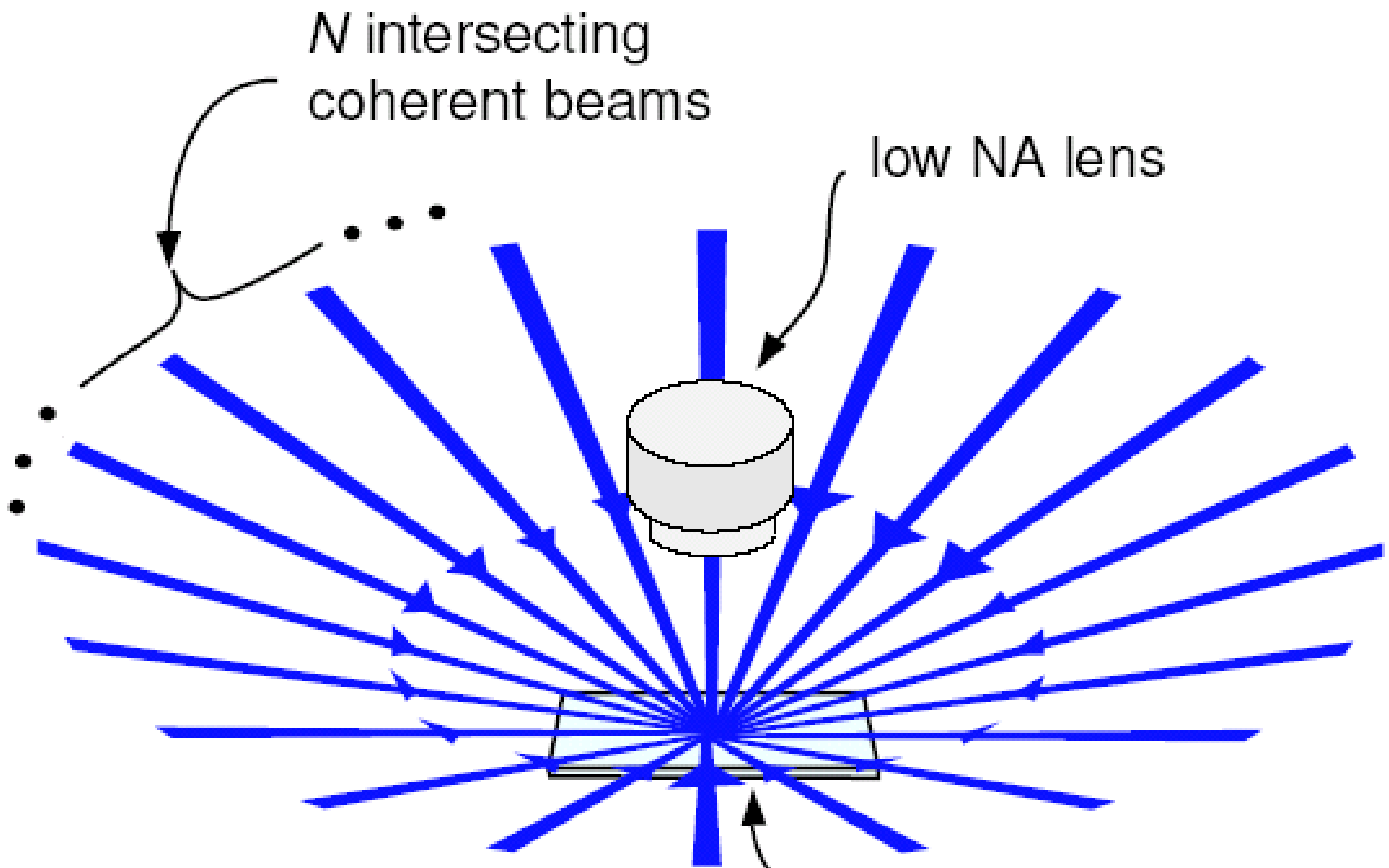
- Precision illumination — Simple imaging
- Multiple images — Textured illumination

- Image detail in response to textures
- Non-uniform samples in FT space

SAM M6

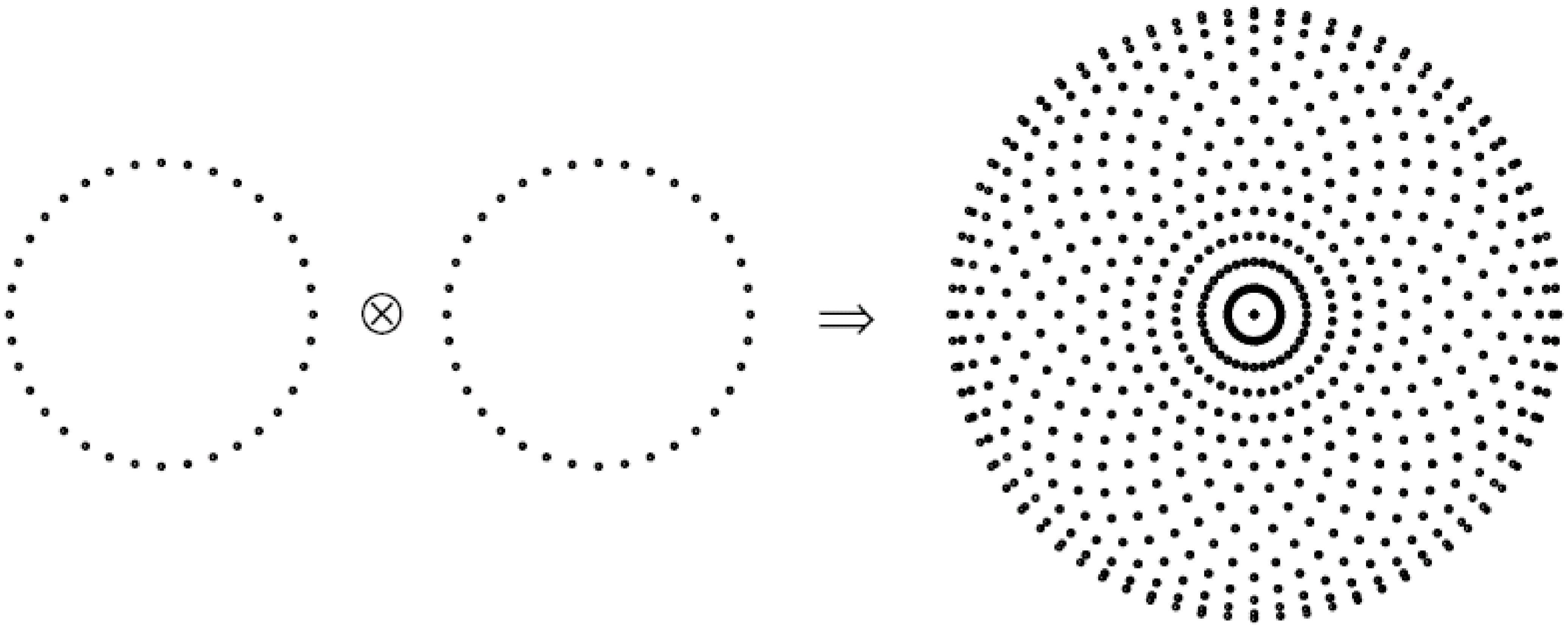


Creating Interference Pattern

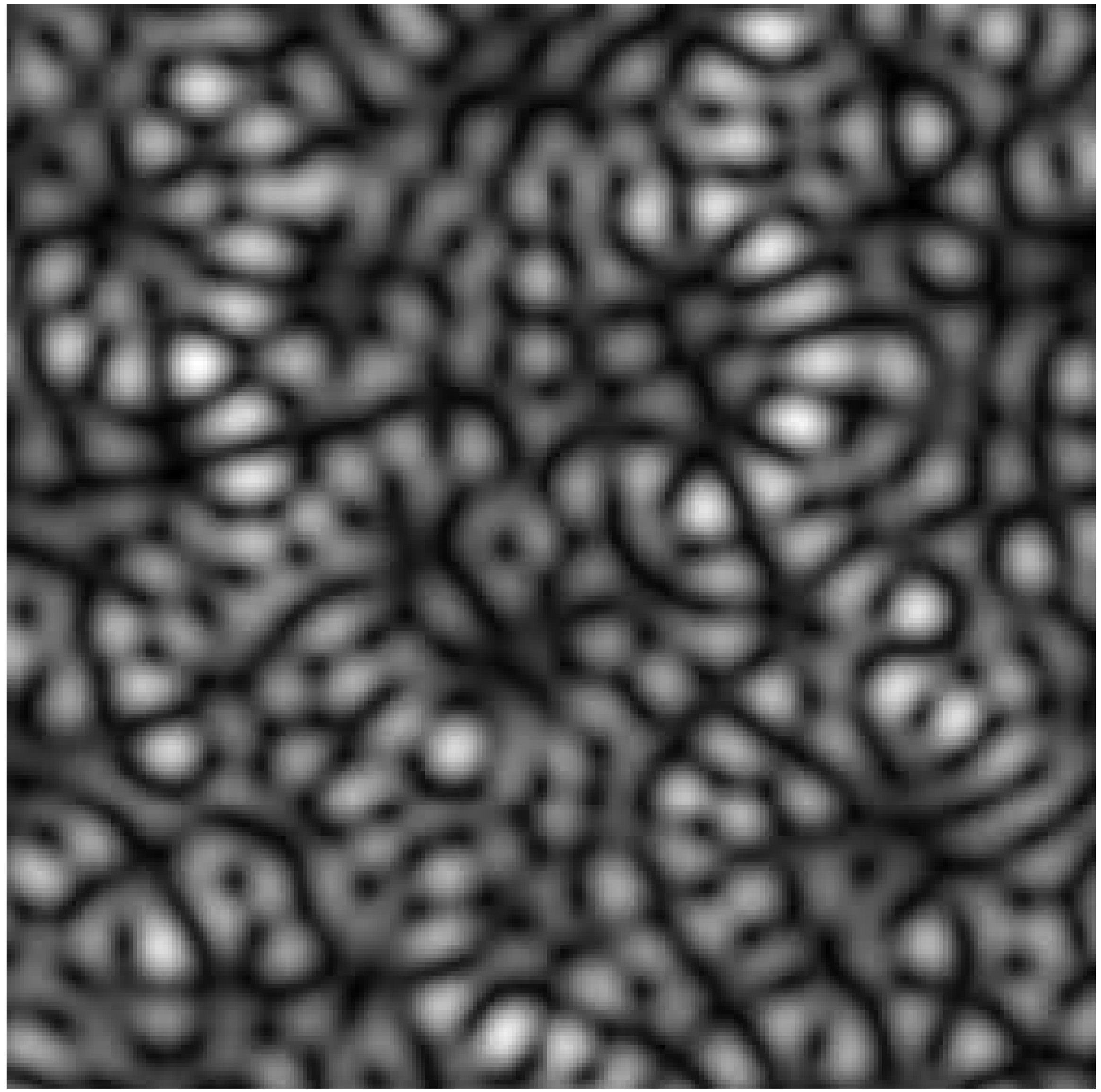


Creating Interference Pattern

Fourier Transform of Texture Pattern



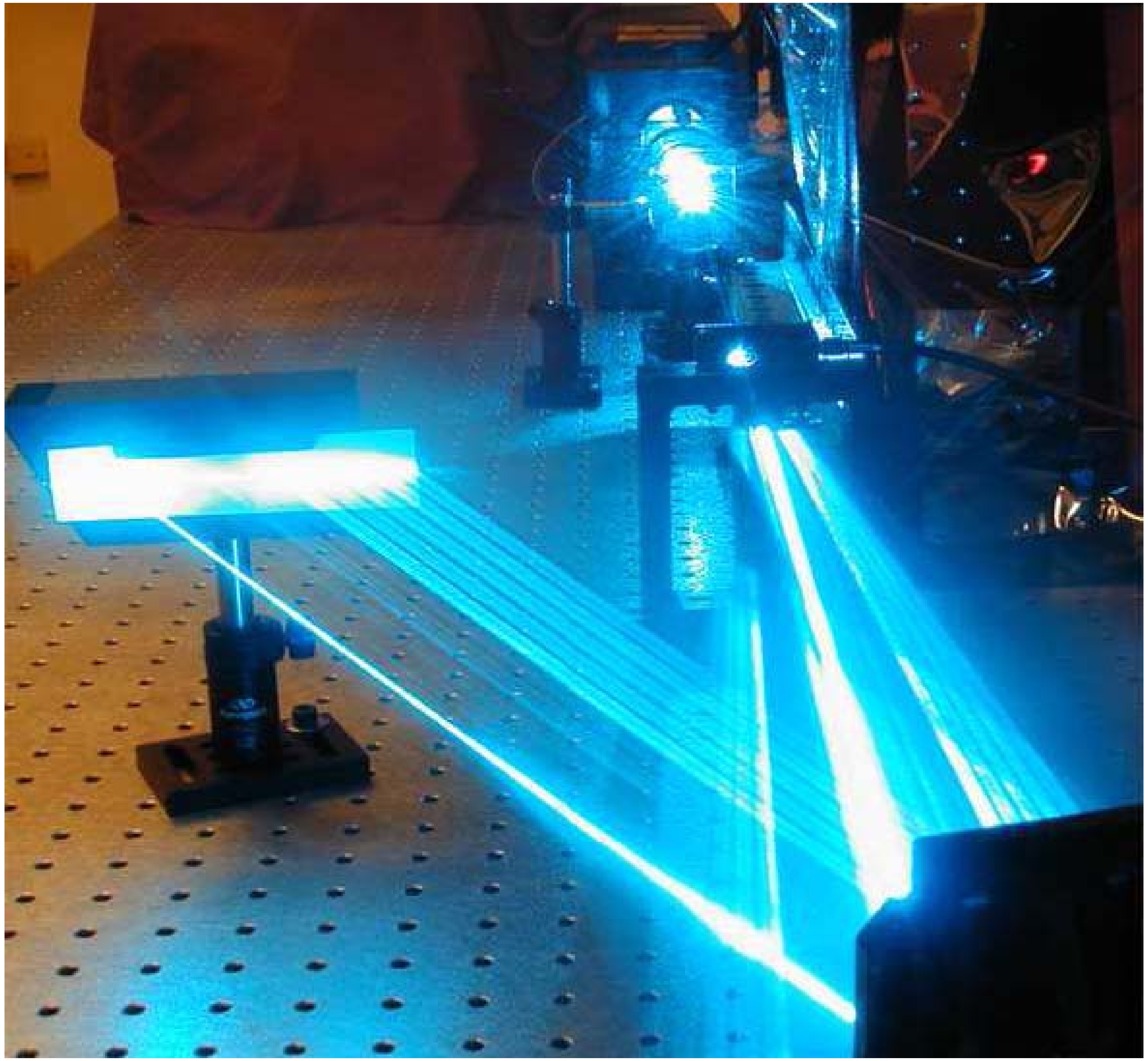
Interference Pattern Texture



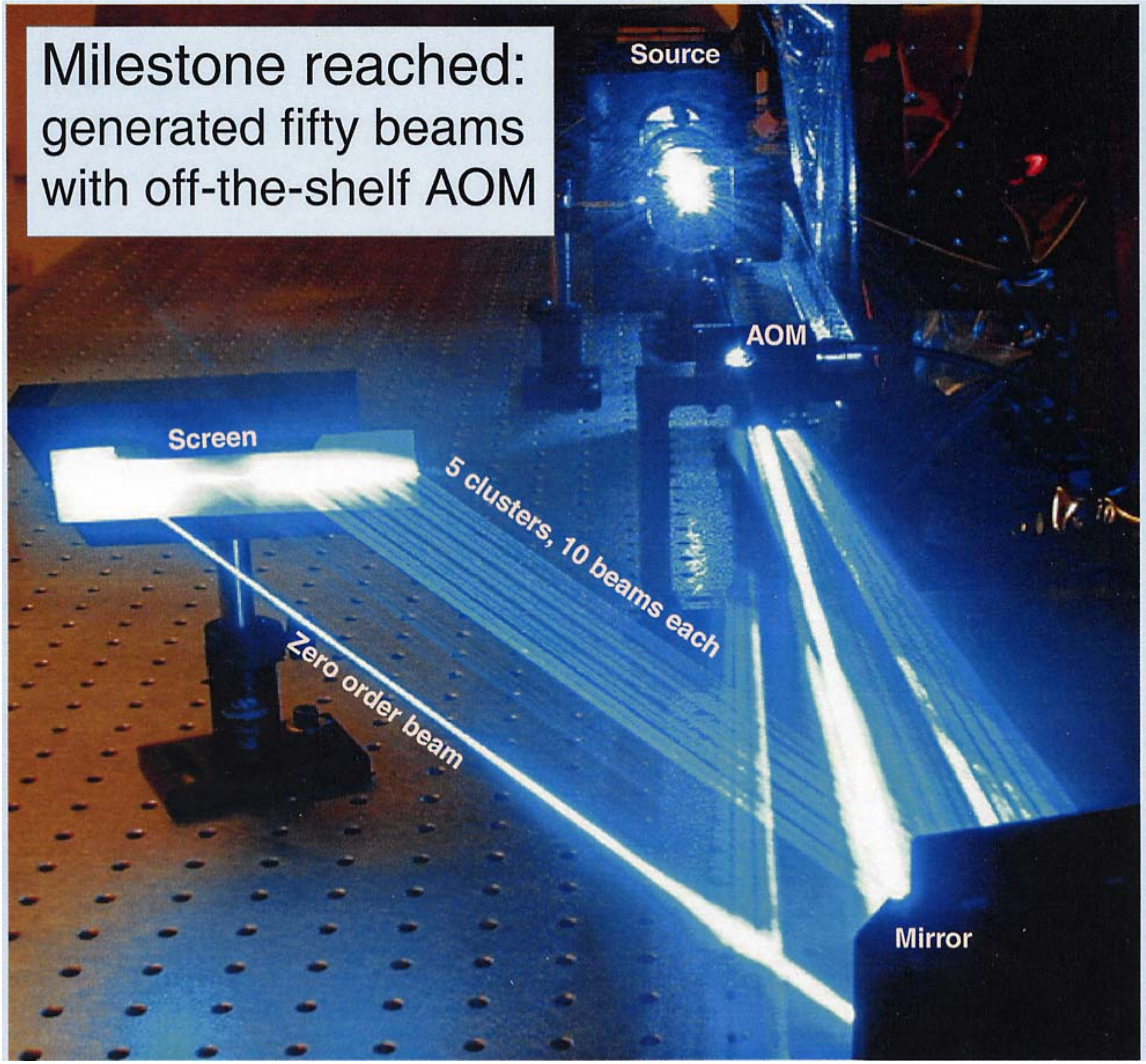
Synthetic Aperture Microscopy

- Interference of many Coherent Beams
- Amplitude and Phase Control of Beams

Amplitude and Phase Control



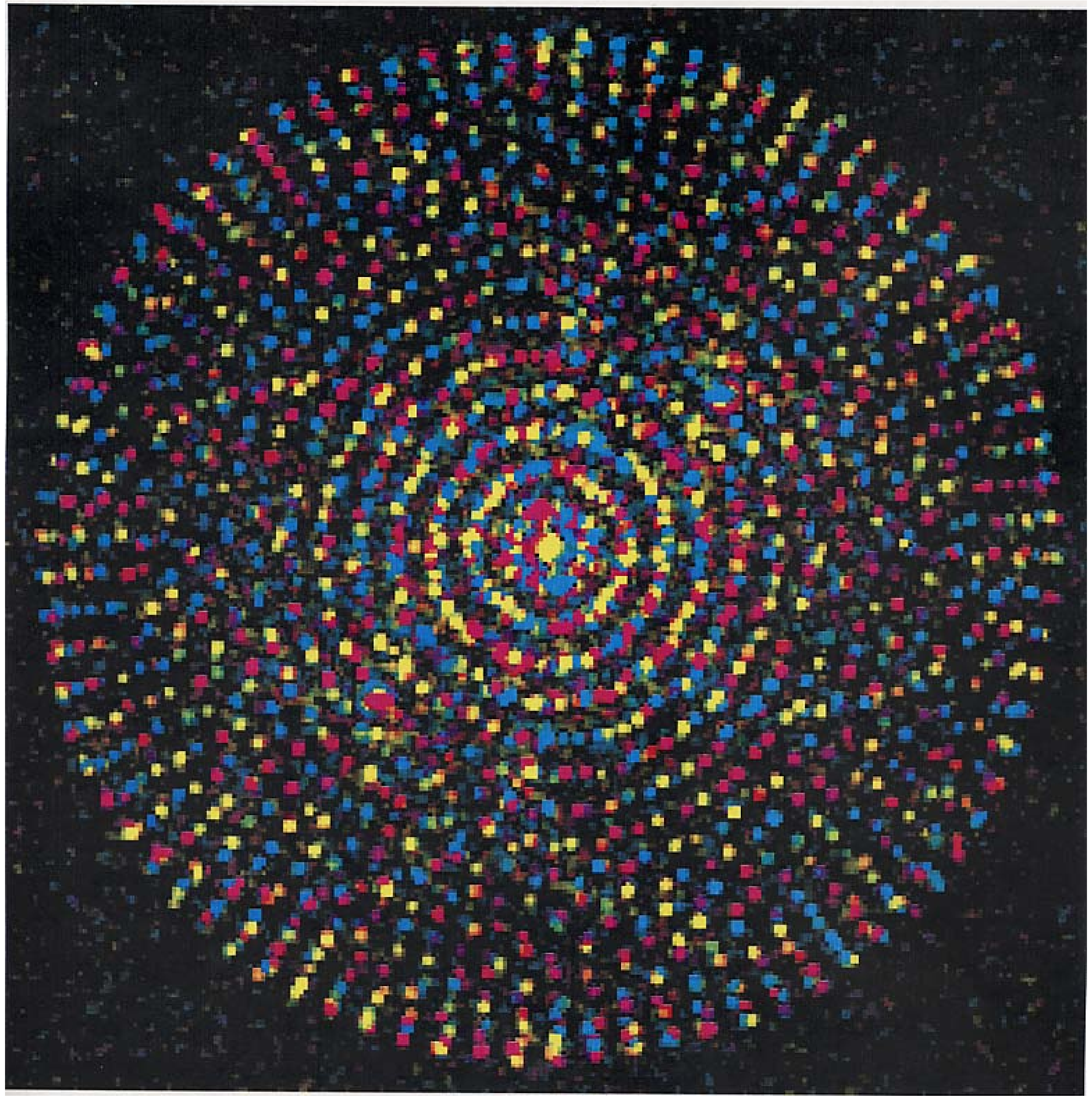
Amplitude and Phase Control



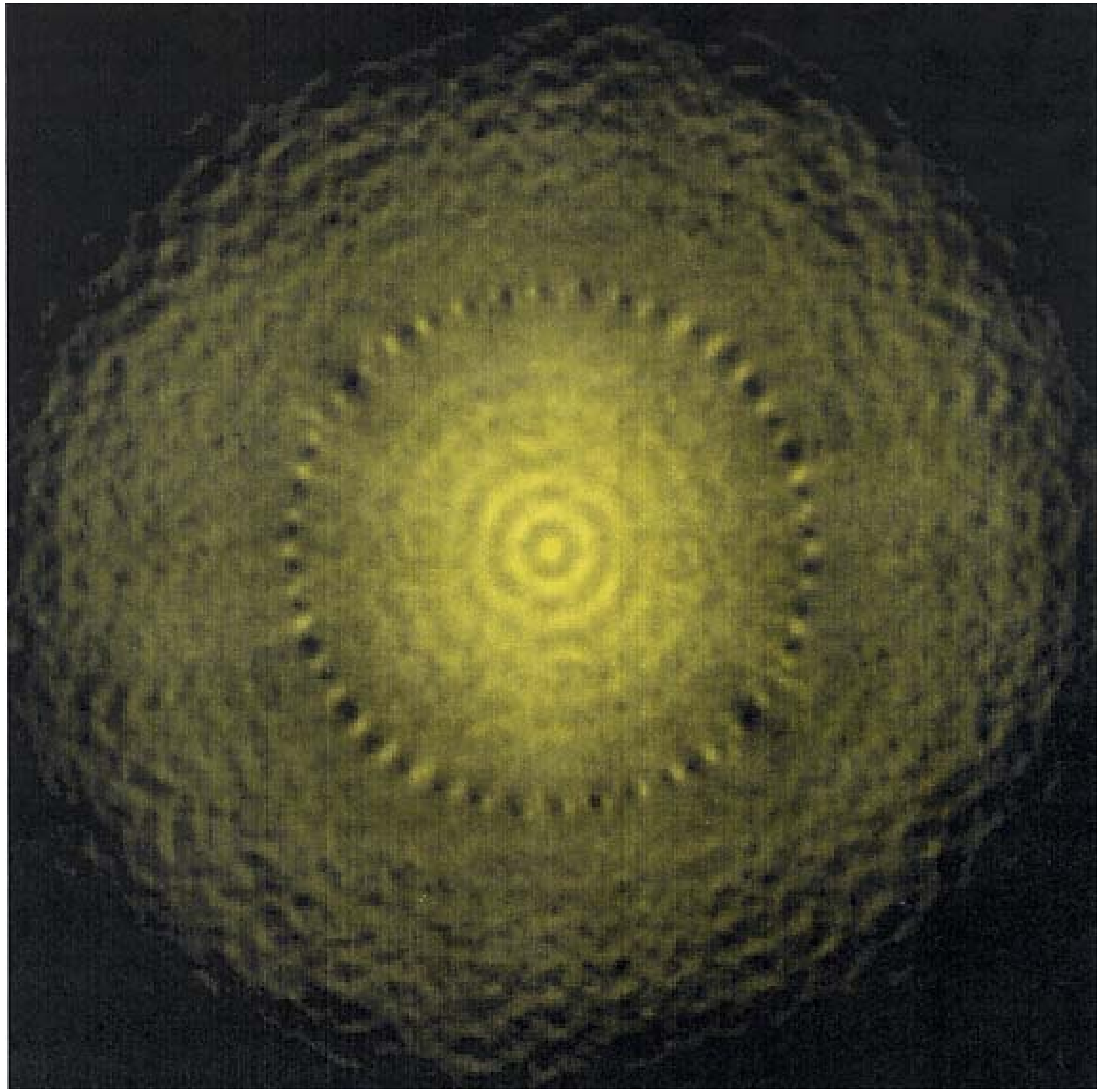
Synthetic Aperture Microscopy

- Interference of many Coherent Beams
- Amplitude and Phase Control of Beams
- On the fly calibration
- Non-uniform inverse FT Least Squares

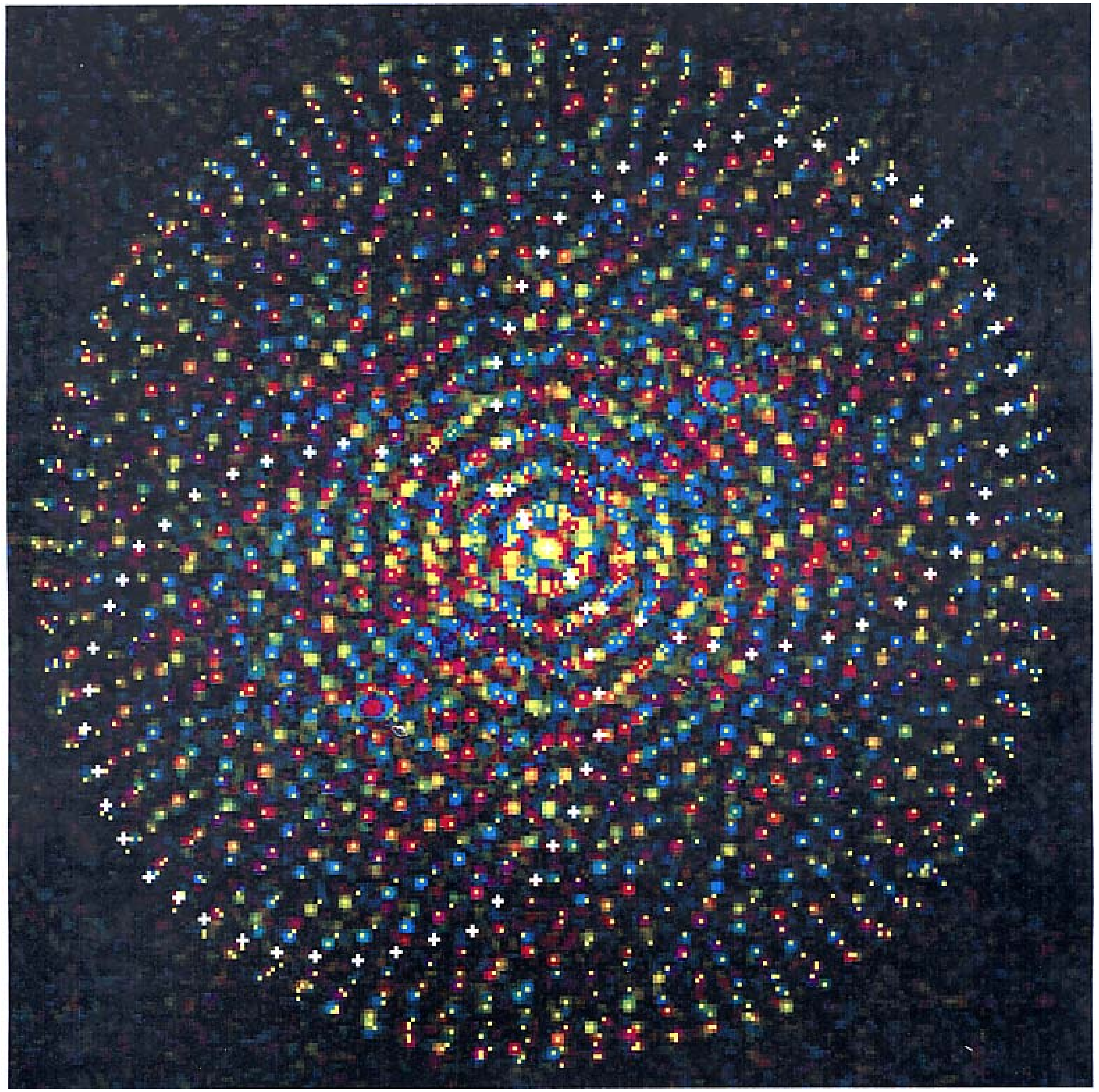
Wavenumber Calibration using FT



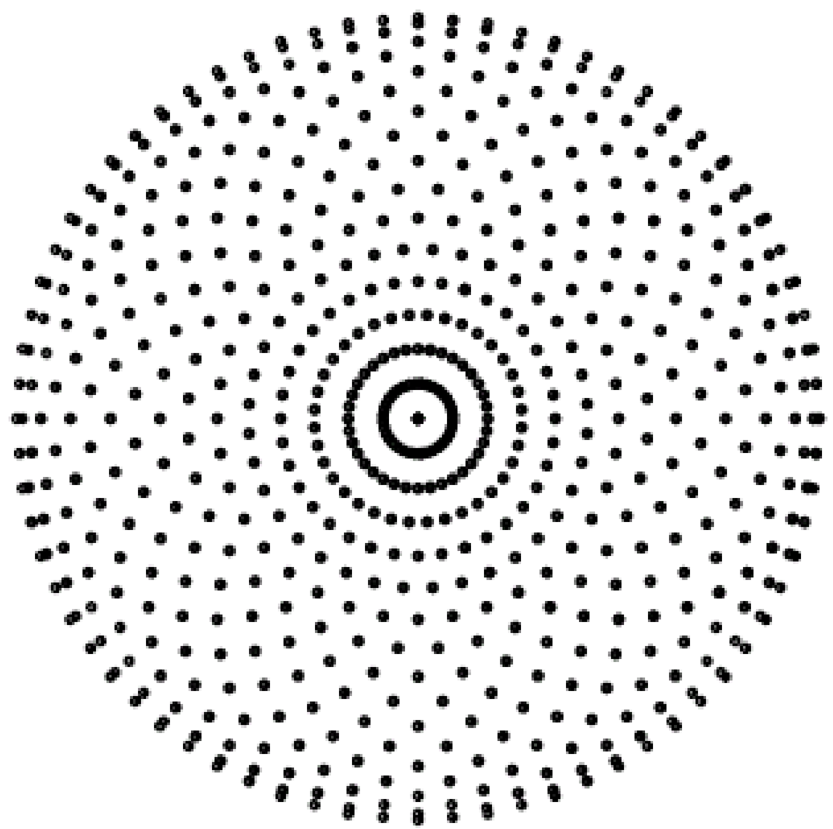
Hough Transform Calibration



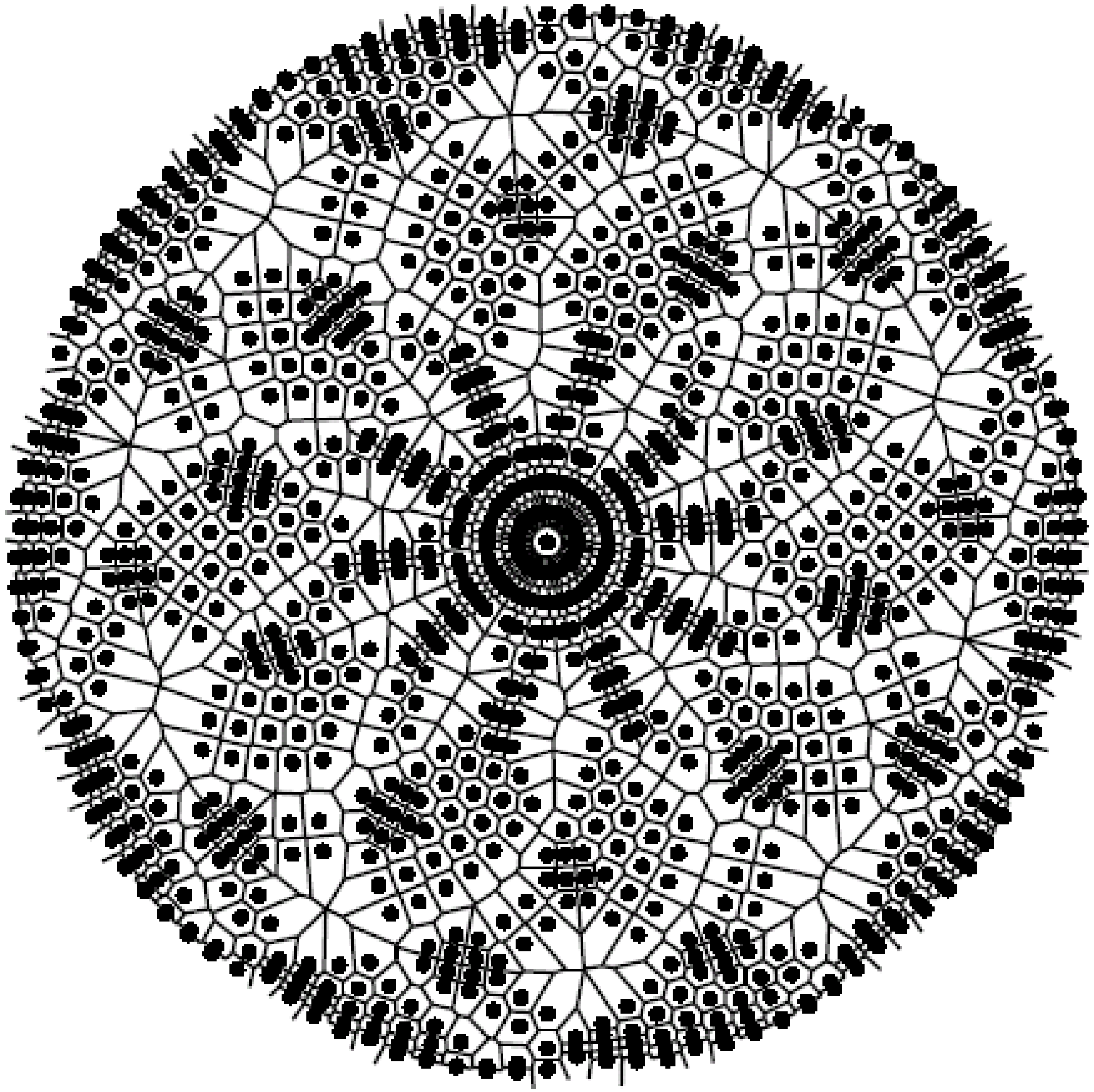
Least Squares Match in FT



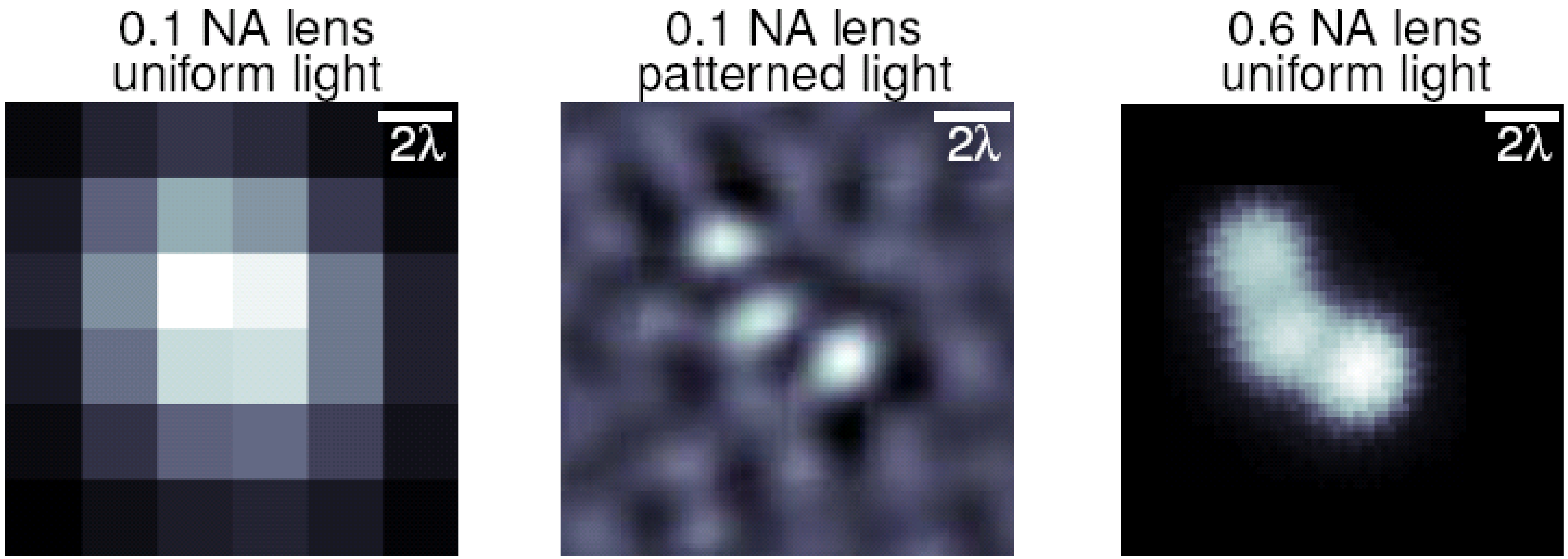
Fourier Transform of Texture Pattern



Uneven Fourier Sampling



Polystyrene Micro Beads (1μm)



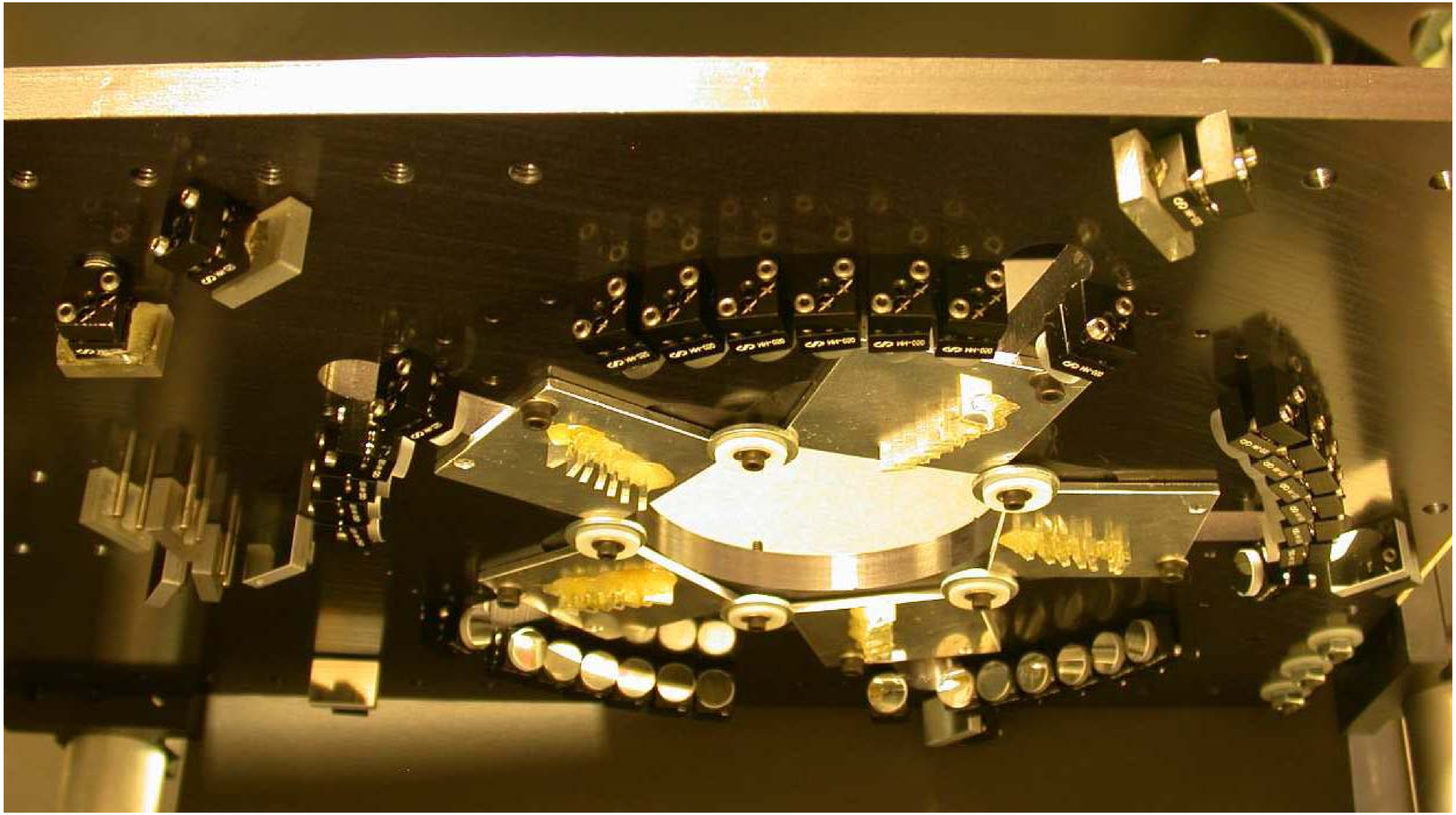
(a) (b) (c)

Resolution Enhancement

- Reflective Optics Illumination

  Vaccum UV — Short Wavelength

Reflective Optics M6



Resolution Enhancement

- Reflective Optics Illumination

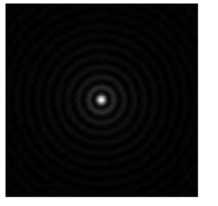
  Vaccum UV — Short Wavelength

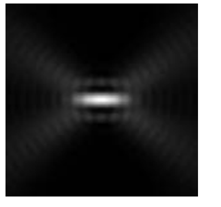
- Fluorescence Mode

  Resolution Determined by Illumination

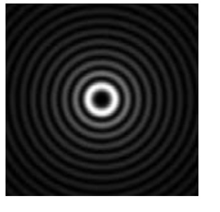
Synthetic Aperture Lithography

- Create pattern — controlled interference

  Example: Two Dots 

  Example: Straight Line 

- Destructive interference “safe zone”

  Example: Bessel Ring 

(2) CODED APERTURE IMAGING

- Can't refract or reflect gamma rays
- Pinhole — tradeoff resolution and SNR

with: Richard Lanza, Roberto Accorsi,
Klaus Ziock, and Lorenzo Fabris.

Coded Aperture Imaging

- Can’t refract or reflect gamma rays
- Pinhole — tradeoff resolution and SNR

- Multiple pinholes
- Complex masks can “cast shadows”

Masks — Fresnel Camera

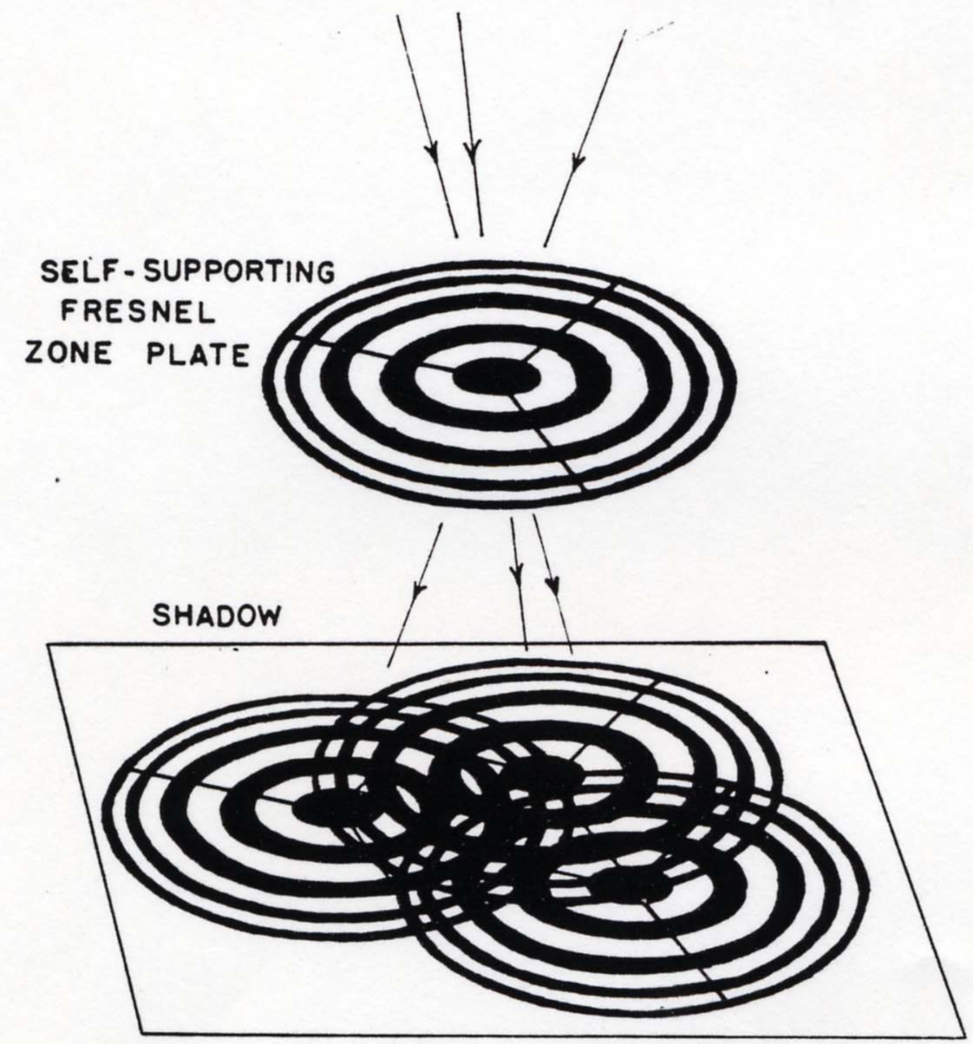
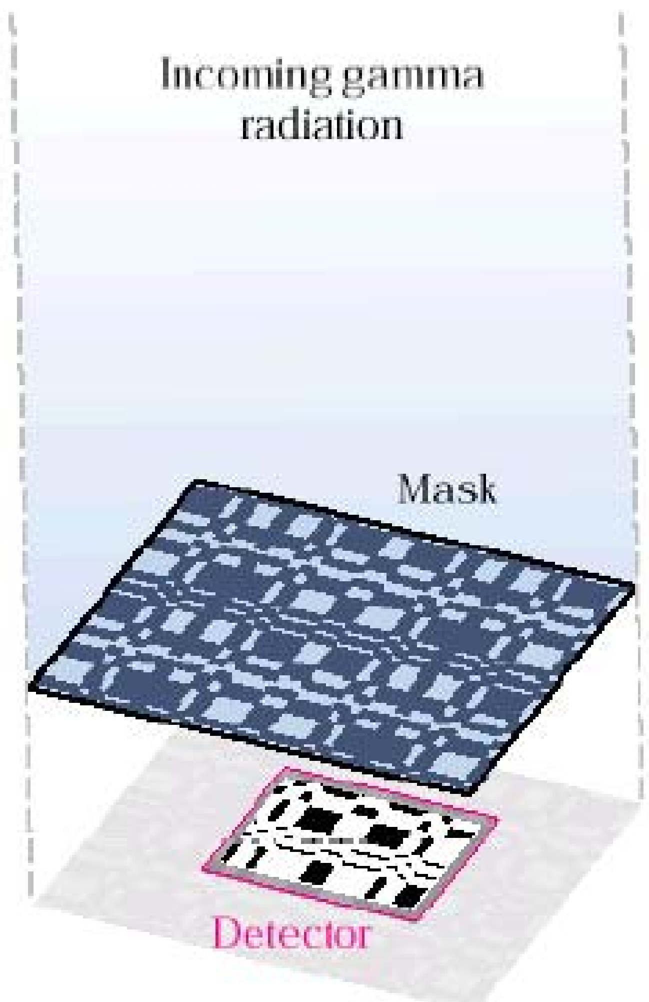
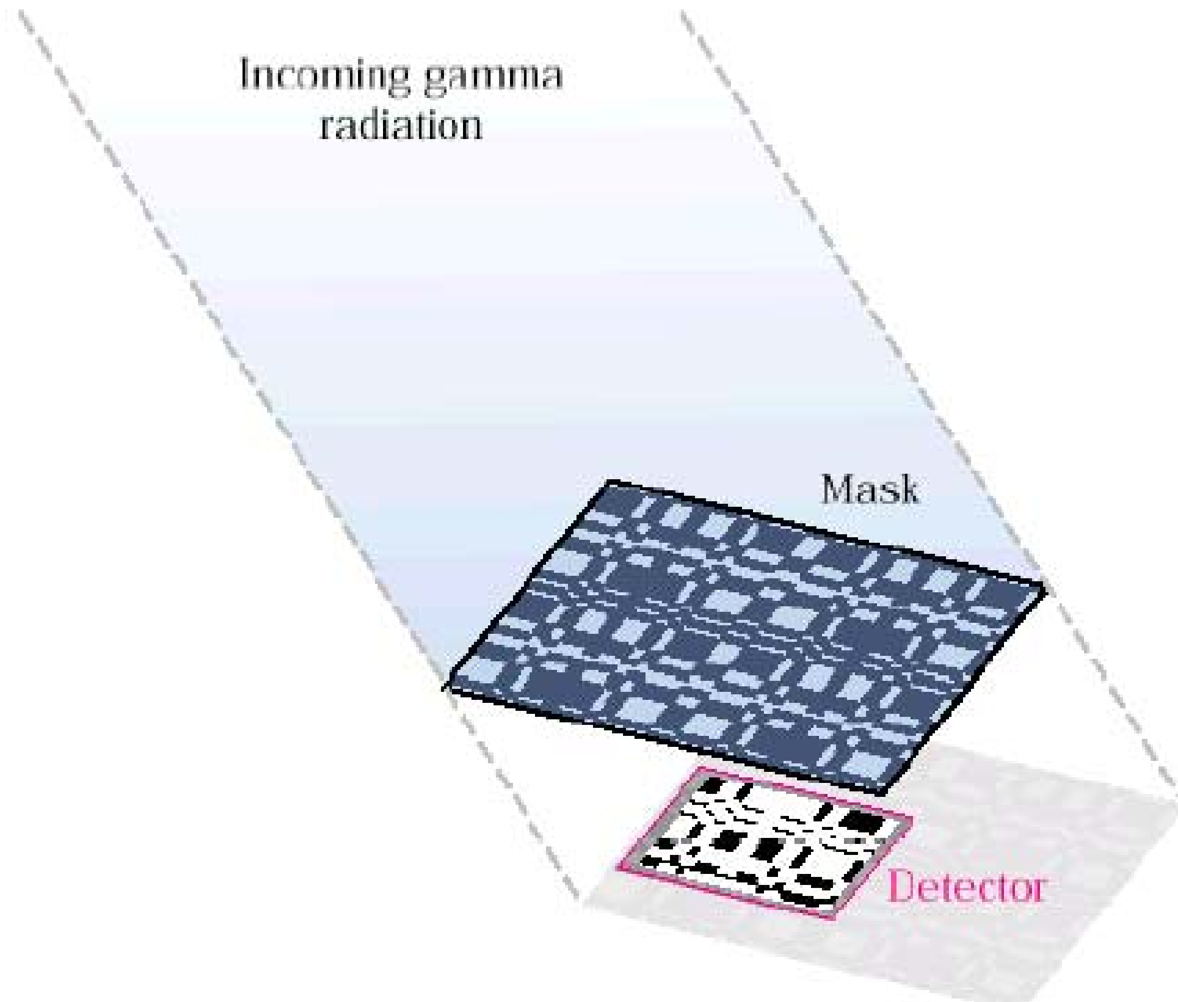


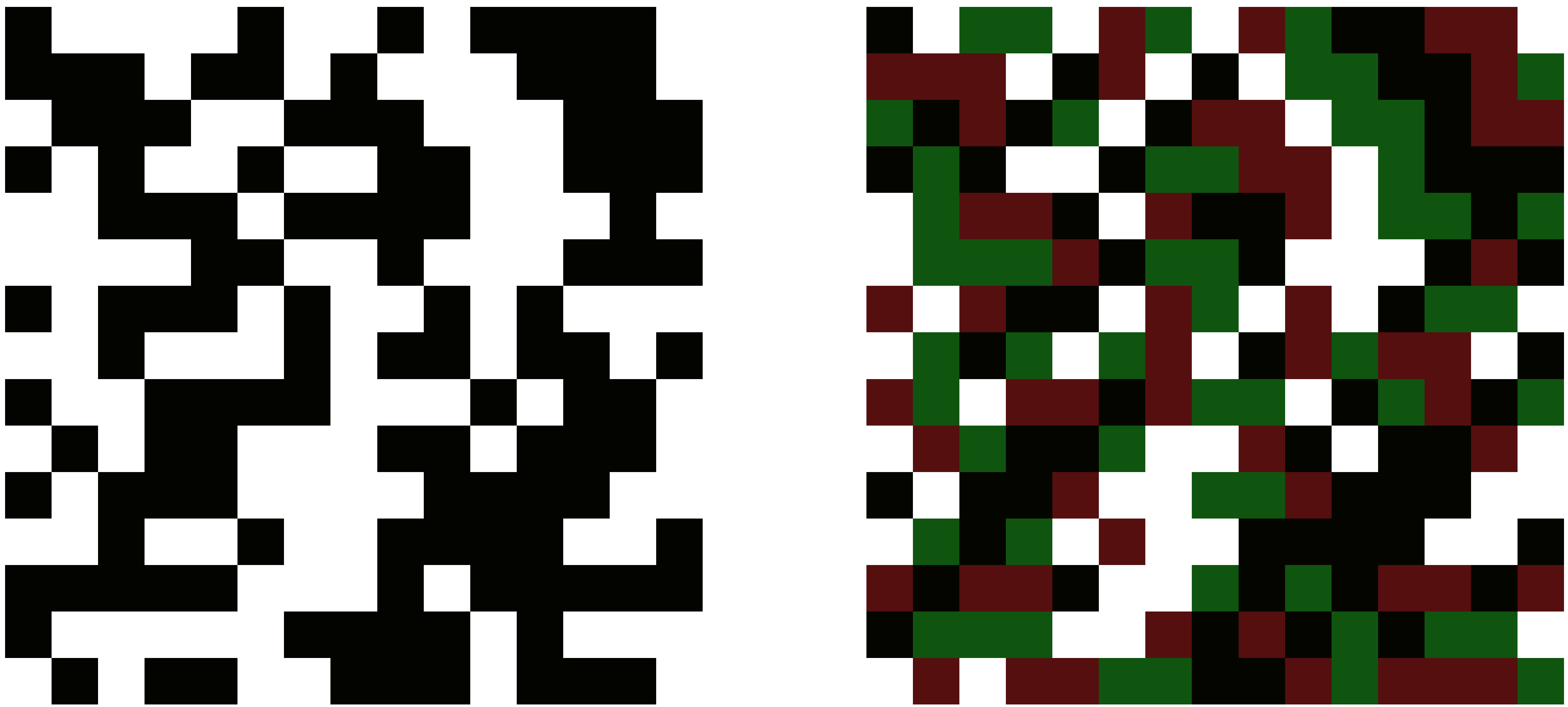
FIG. 1. X-ray star camera.

Coded Aperture Principle





Decoding Method Rationale

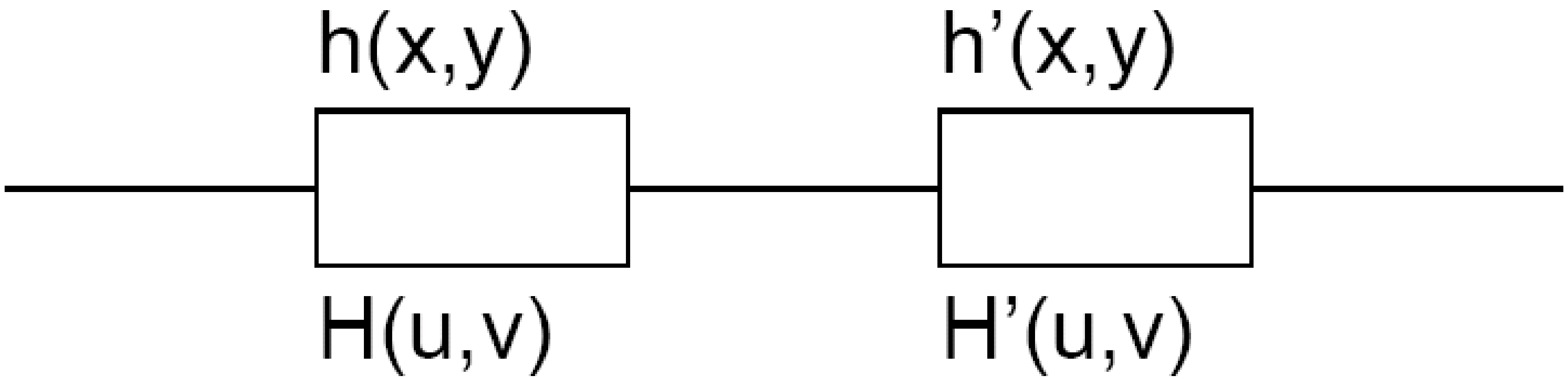


Coded Aperture Imaging

- Can’t refract or reflect gamma rays
- Pinhole — tradeoff resolution and SNR
- Complex masks can “cast shadows”

- Decoding by Correlation
- Special Masks with Flat Power Spectrum

Mask Design — Inverse Systems



$$h(x,y) \otimes h'(x,y) = \delta(x,y)$$

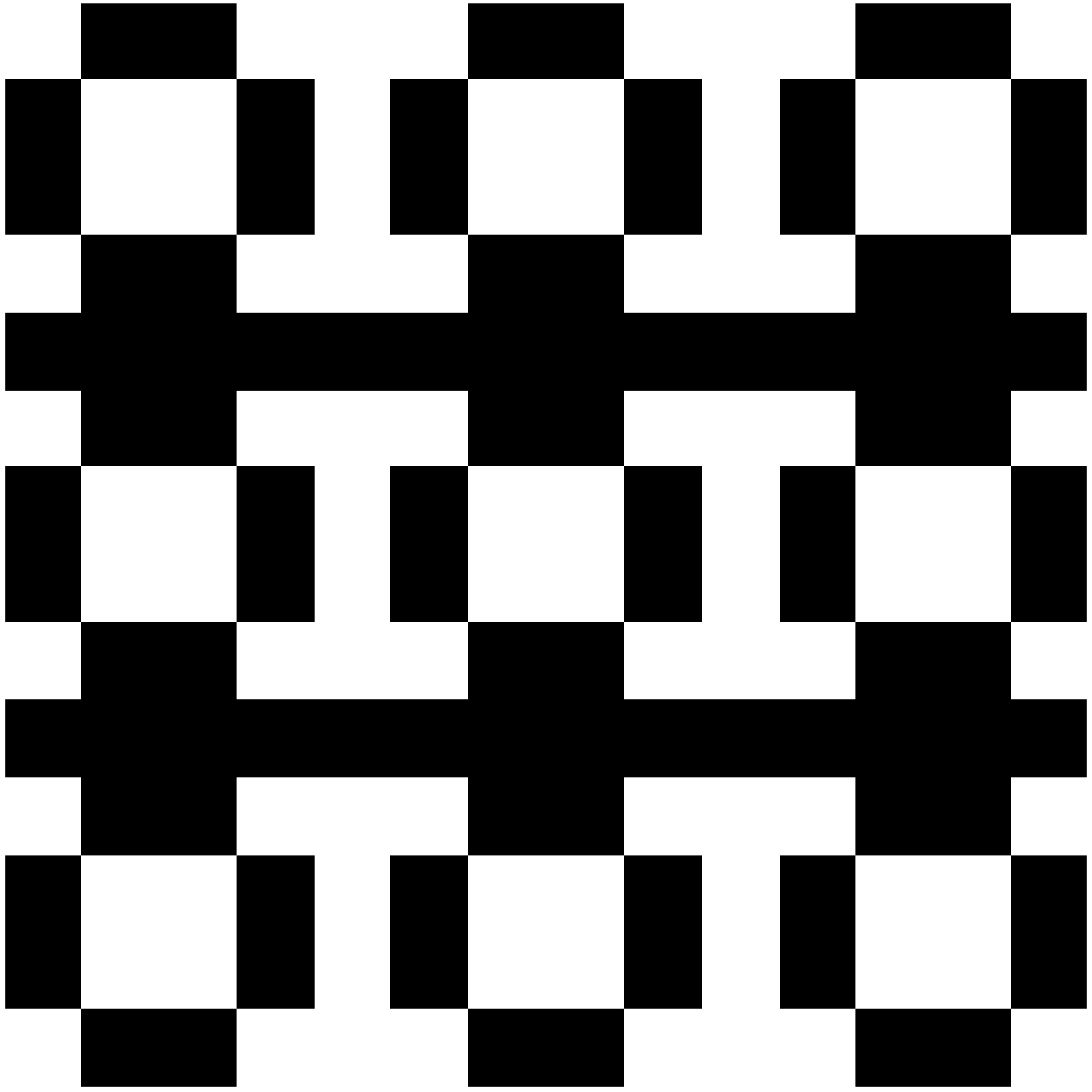
$$H(u,v)\ \ H'(u,v) = 1$$

Maximizing SNR

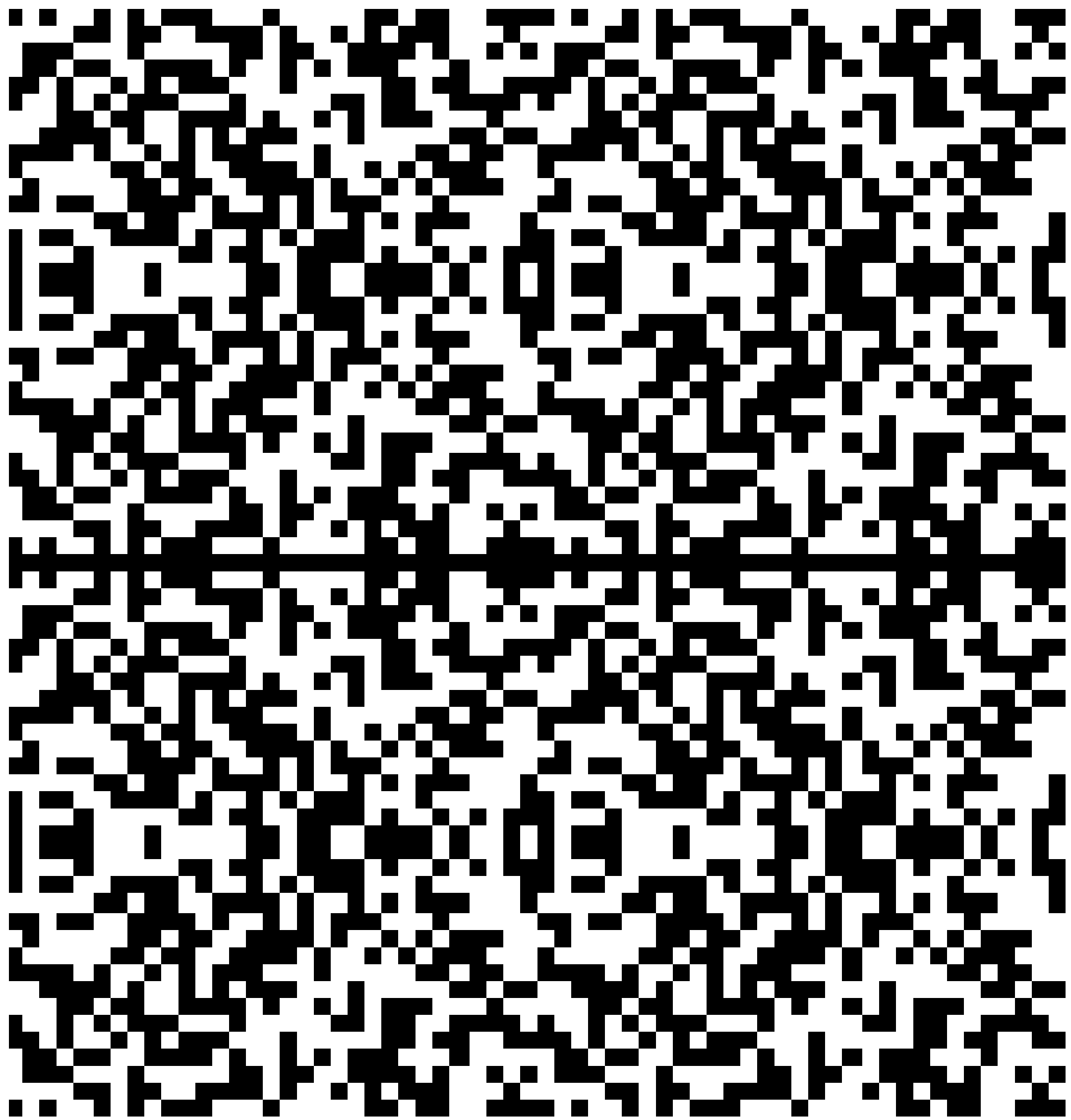
$$\min \sum_{i=1}^{n} w_i^2 \qquad \text{subject to} \qquad \sum_{i=1}^{n} w_i = 1$$

$$\text{yields} \qquad w_i = \frac{1}{n}$$

Masks — Legri URA



Masks — XRT Coarse



Mask Design — 1D

Definition: q is a quadratic residue $\pmod p$ if $\exists\, n$ s.t. $n^2 \equiv q \pmod p$

Legendre symbol

$$\left(\frac{a}{p}\right) = \begin{cases} 1 & \text{if } a \text{ is quadratic residue} \\ -1 & \text{otherwise} \end{cases}$$

Correlation with zero shift $(p-1)/2$
Correlation with non-zero shift $(p-1)/4$

Mask Design

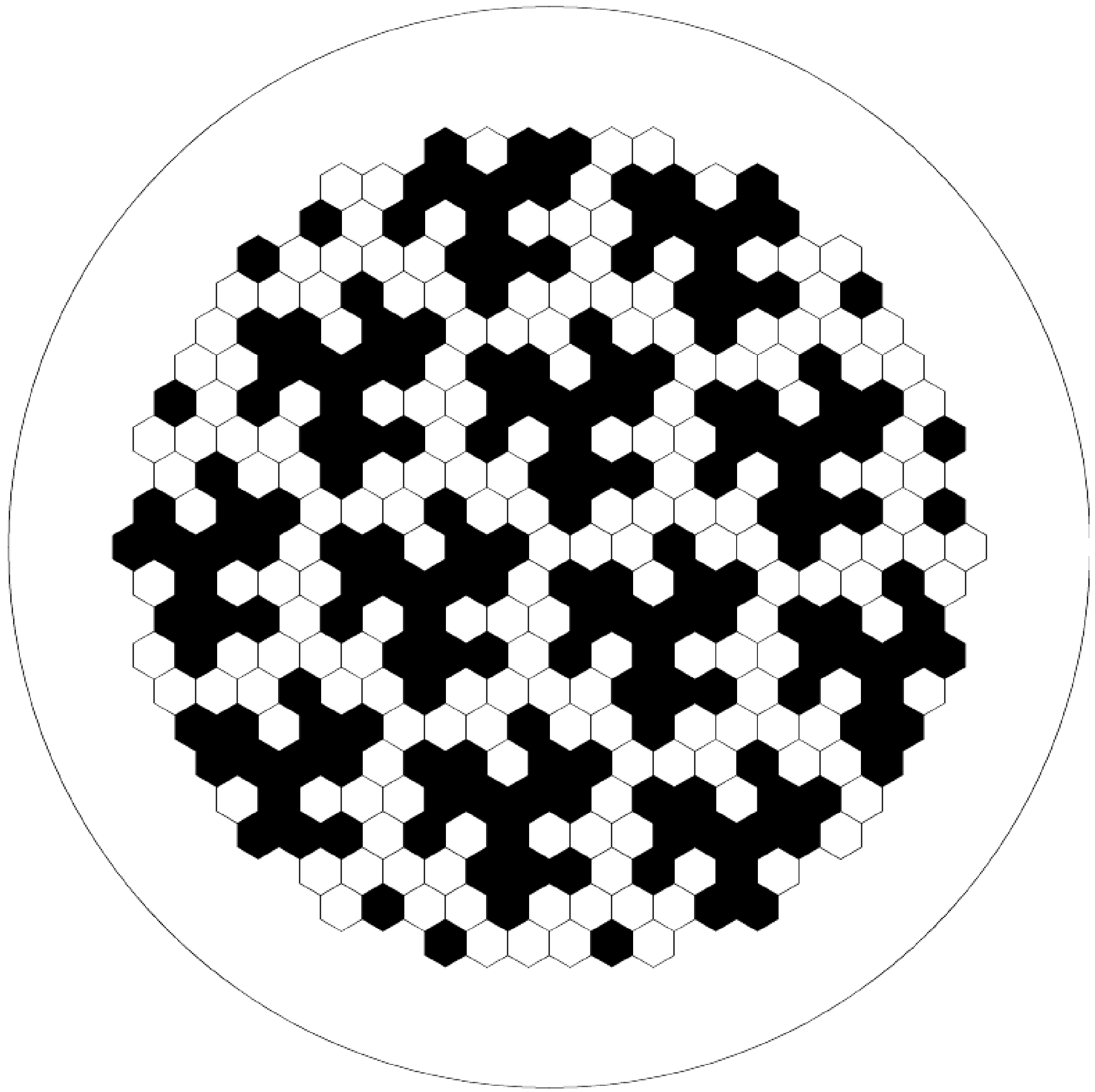
- Auto Correlation

$$a(i) = \frac{(p-1)}{4}(1 + \delta(i))$$

- Power Spectrum

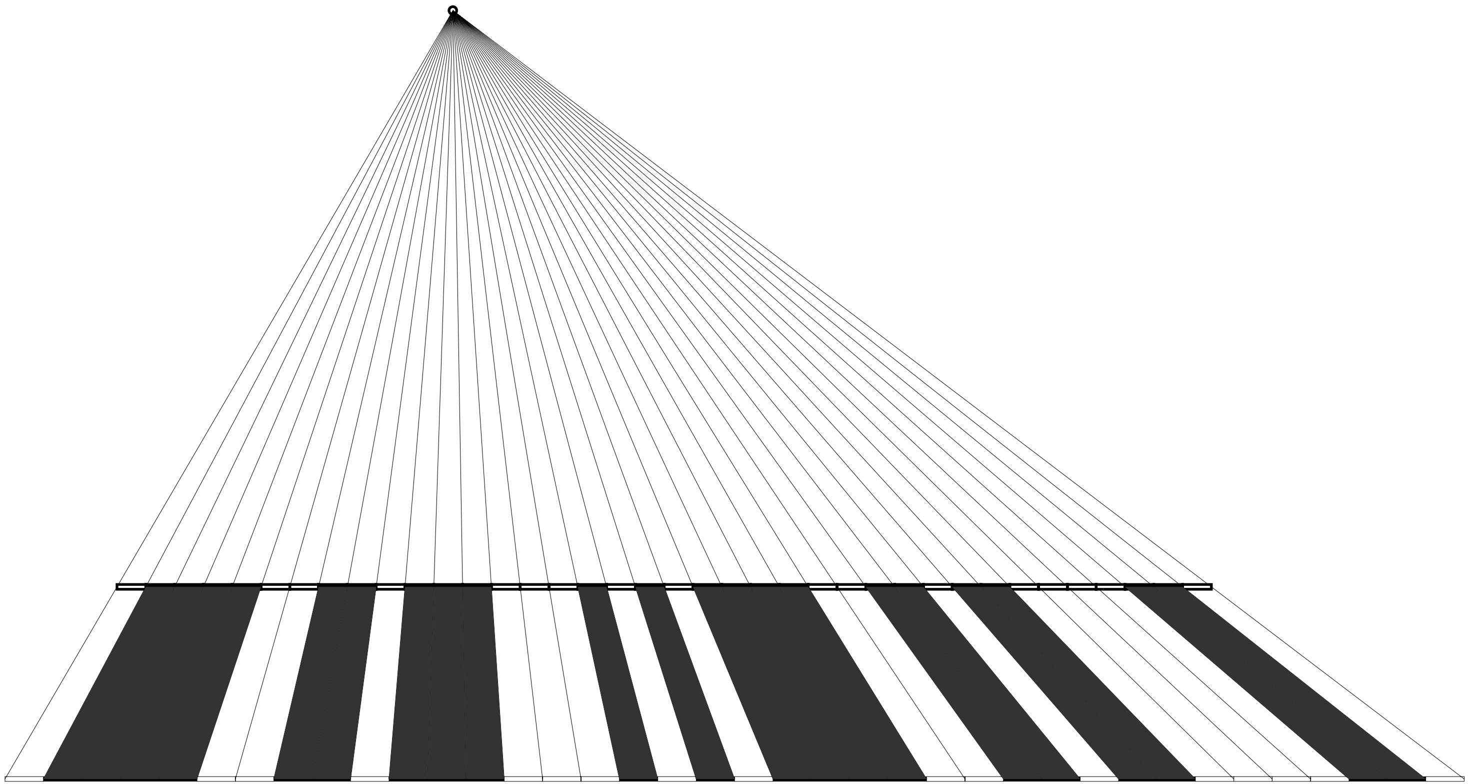
$$A(j) = \frac{(p-1)}{4}(\delta(j) + 1)$$

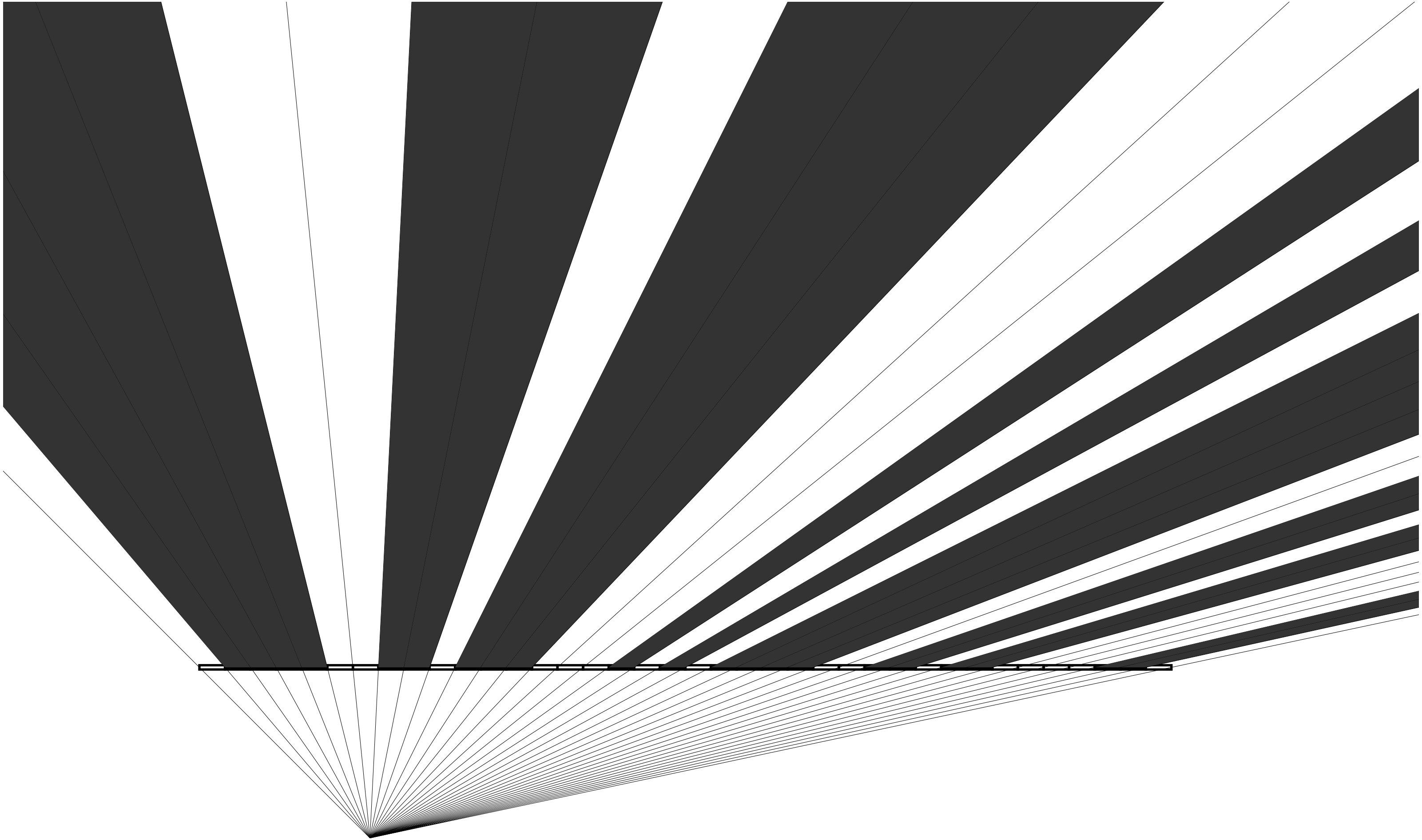
Masks — Hexagonal



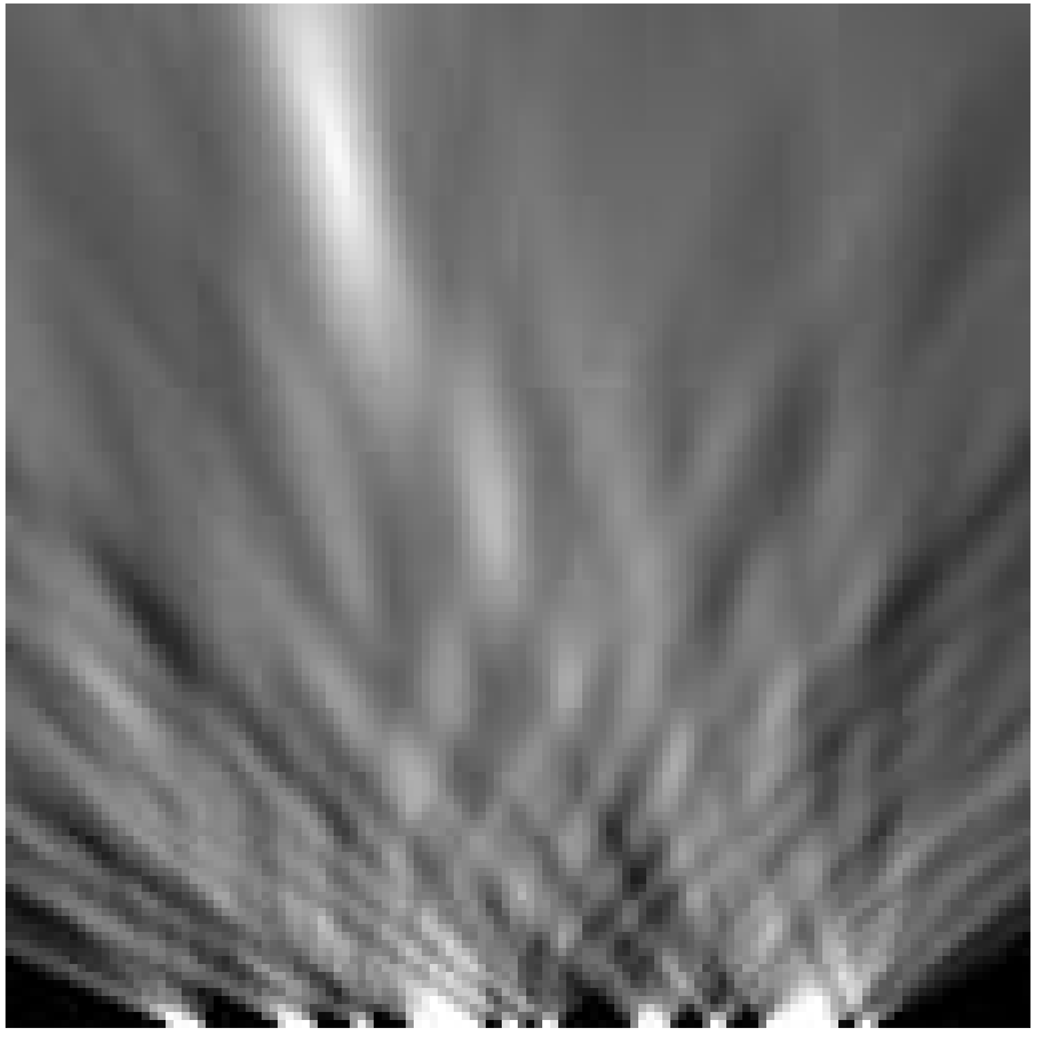
Coded Aperture Extensions

- Artifacts due to Finite Distance
- Mask / Countermask Combination





Coded Aperture Backprojection



Reconstruction Animation

Coded Aperture Extensions

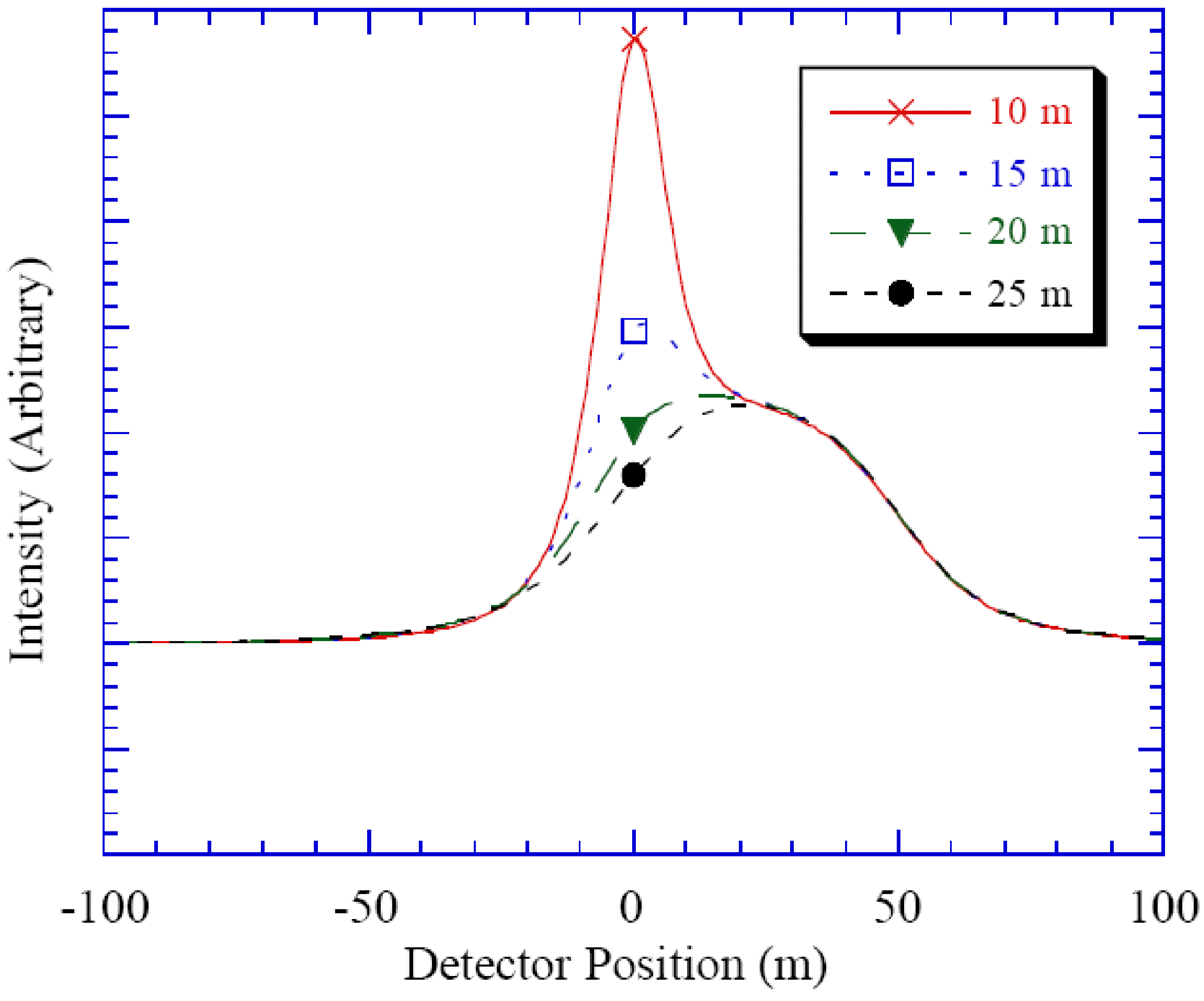
- Artifacts due to Finite Distance
- Mask / Countermask Combination

- Multiple Detector Array Positions
- “Synthetic Aperture” radiography

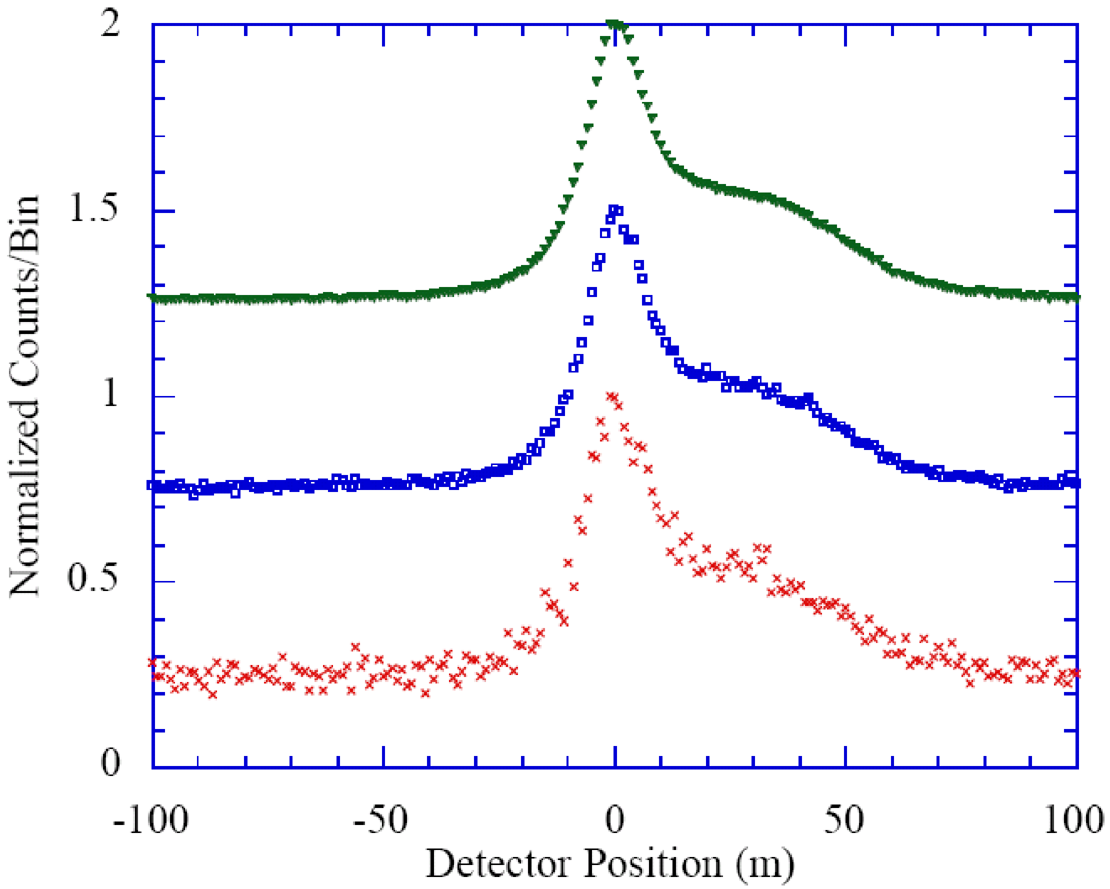
Coded Aperture Applications

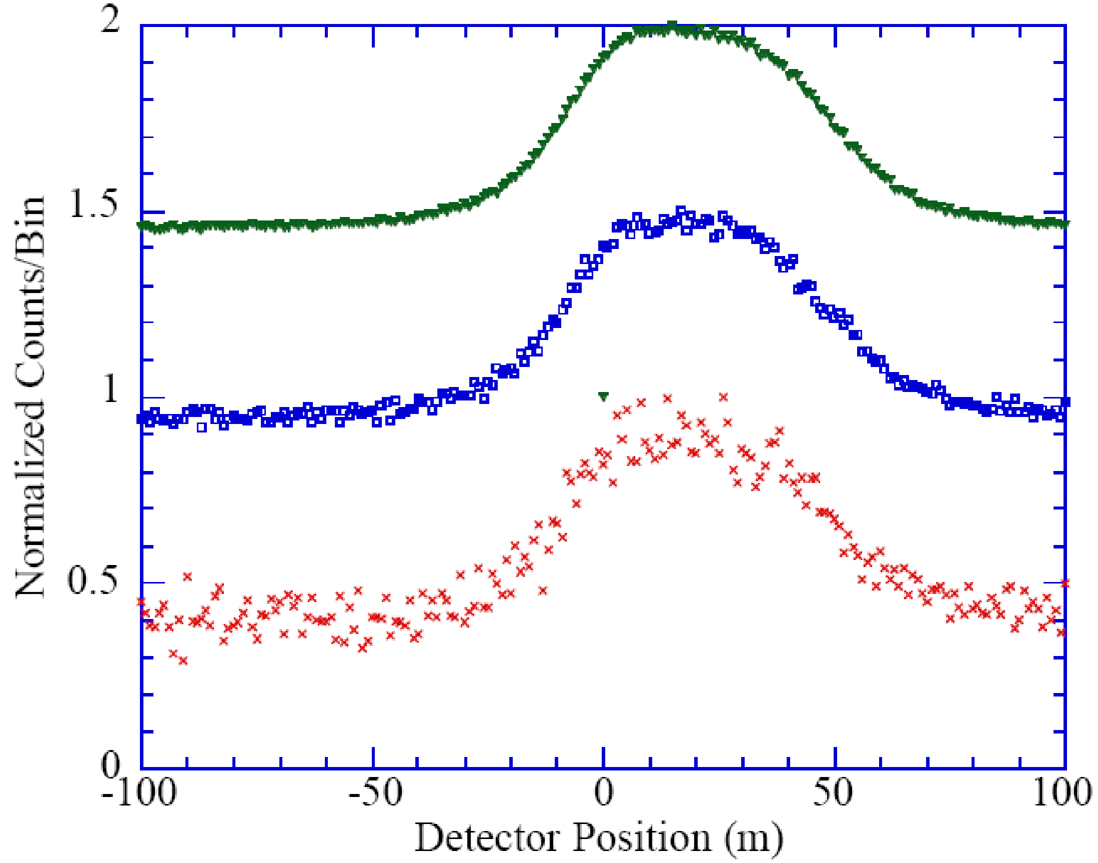
- Detection of Fissile Material
- Large Area Detector Myth
- Signal *and* Background Amplified

Spatially Varying Background

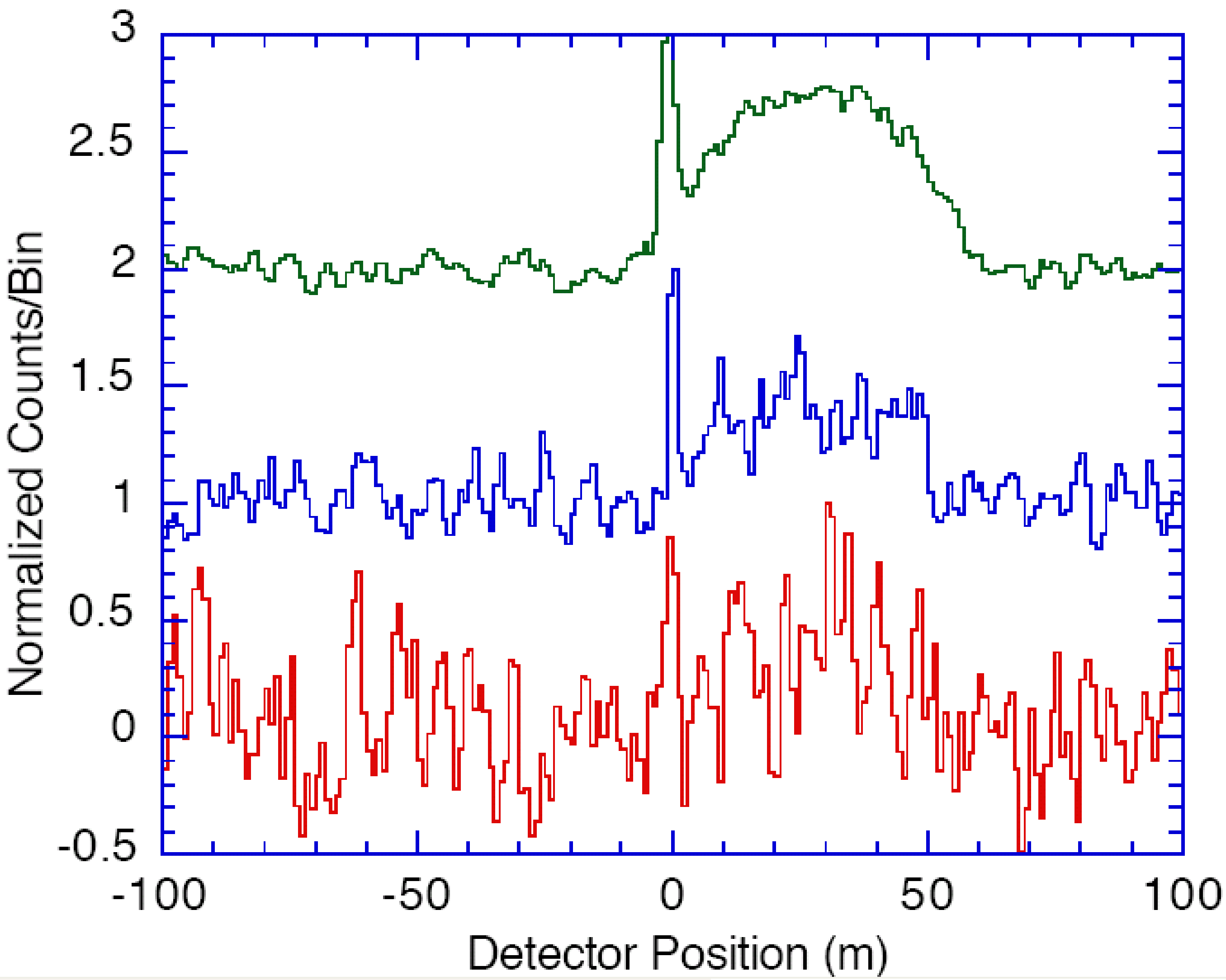


Large Area Alone Doesn't Help

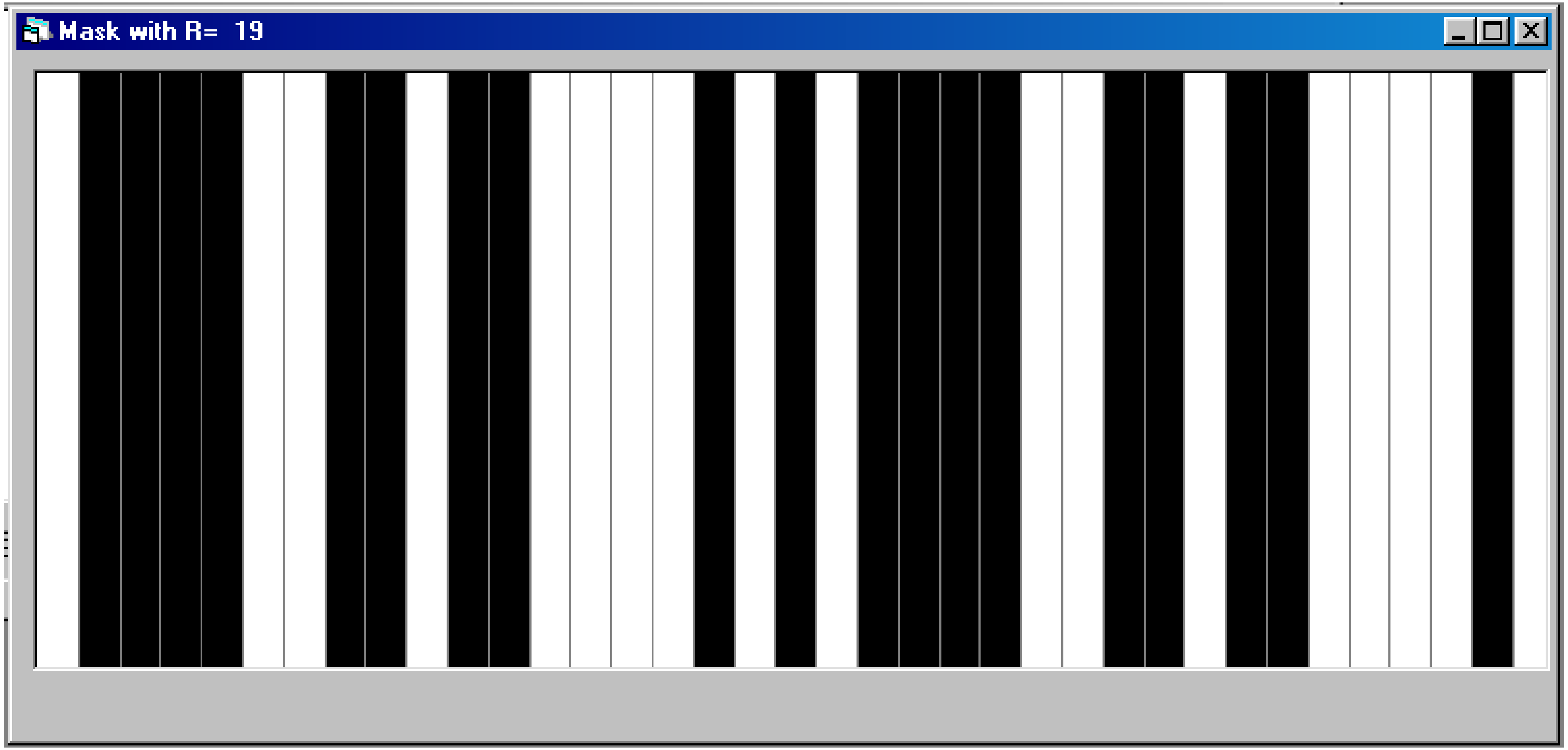




Imaging and Large Area Do!



Coded Aperture Example

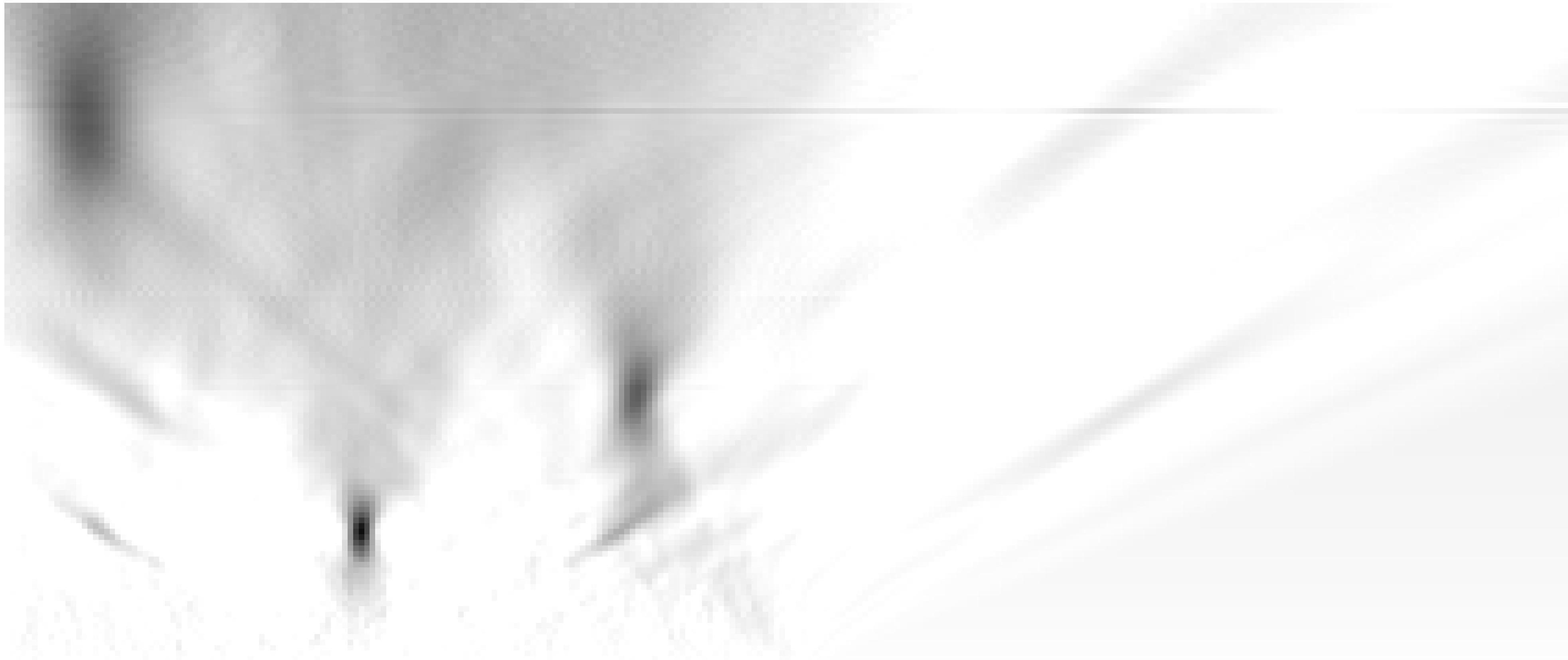


- Imaging — $1/R$ instead of $1/R^2$

Coded Aperture Detector Array

Computational Imaging System

Coded Aperture Example



Three weak, distant radioactive sources

Reconstruction Animation

Coded Aperture Applications

- Detection of Fissile Material
- Imaging — $1/R$ instead of $1/R^2$
- Increasing Gamma Camera Resolution
- Replacing Rats with Mice

(3) DIAPHANOGRAPHY

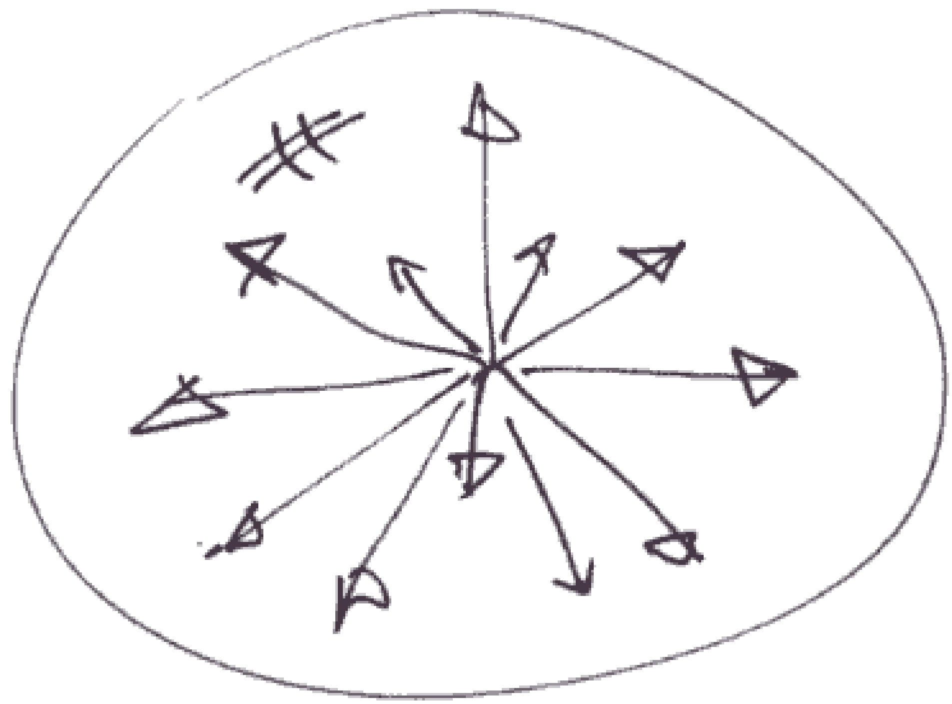
(Diffuse Optical Tomography)

- Highly Scattering — Low Absorption
- Many Sources — Many Detectors

with: Xiaochun Yang, Richard Lanza,
Charles Sodini, and John Wyatt.

Diaphanography

- Randomization of Direction



- *Scalar* Flux Density

Diaphanography

- Approximation: Diffusion Equation

$$\Delta v(x,y) + \rho(x,y)c(x,y) = 0$$

  $v(x,y)$ flux density

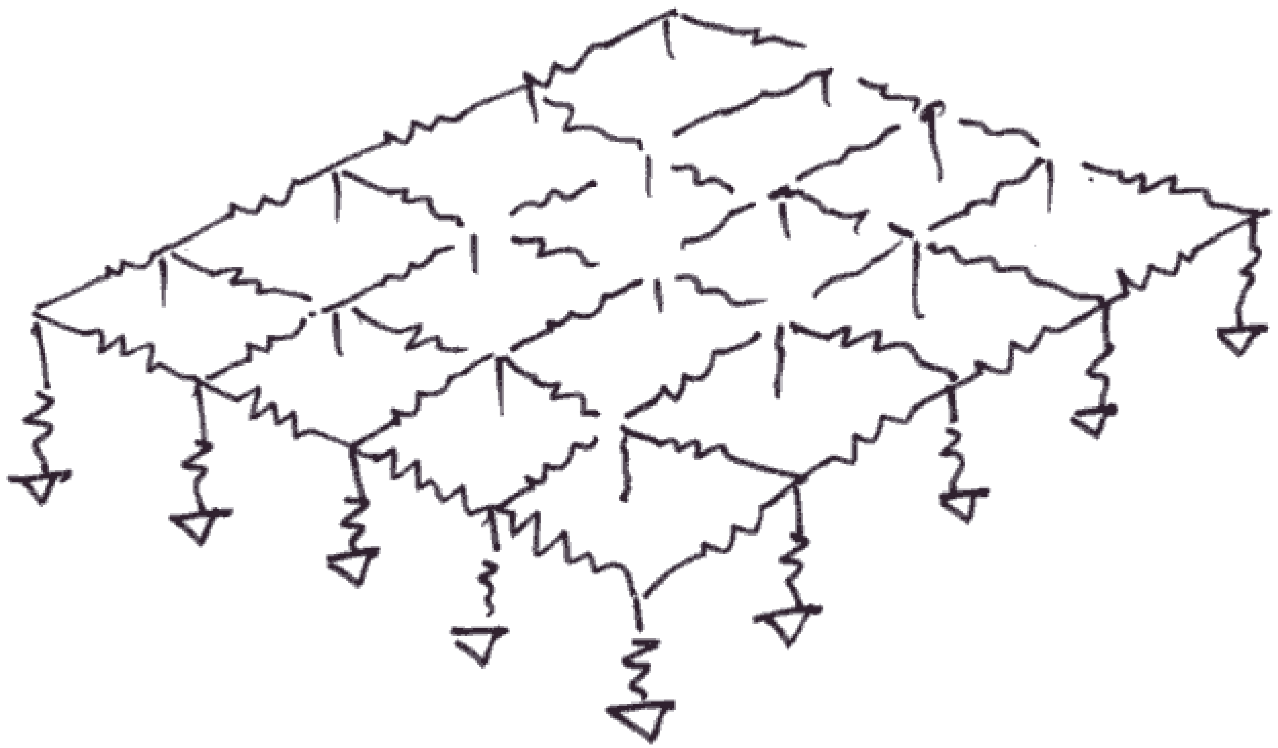
  $\rho(x,y)$ scattering coefficient

  $c(x,y)$ absorption coefficient

- Forward: given $c(x,y)$ find $v(x,y)$

Diaphanography

- Approximation: Diffusion Equation



- Leaky Resistive Sheet Analog (2D)

Diaphanography

- "Invert" Diffusion Equation

- Regions of Influence

(4) EXACT CONE BEAM ALGORITHM

- Faster Scanning—Fewer Motion Artifacts
- Lower Exposure—Uniform Resolution

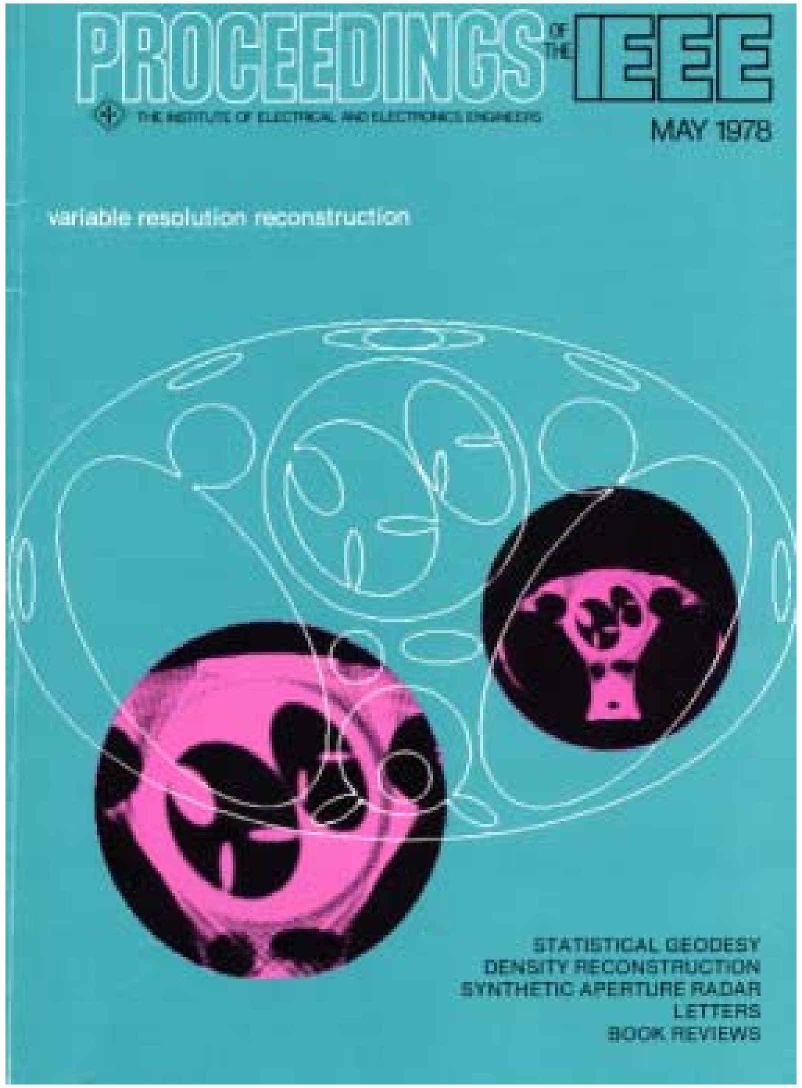
with: Xiaochun Yang

Exact Cone Beam Reconstruction

- Faster Scanning—Fewer Motion Artifacts
- Lower Exposure—Uniform Resolution

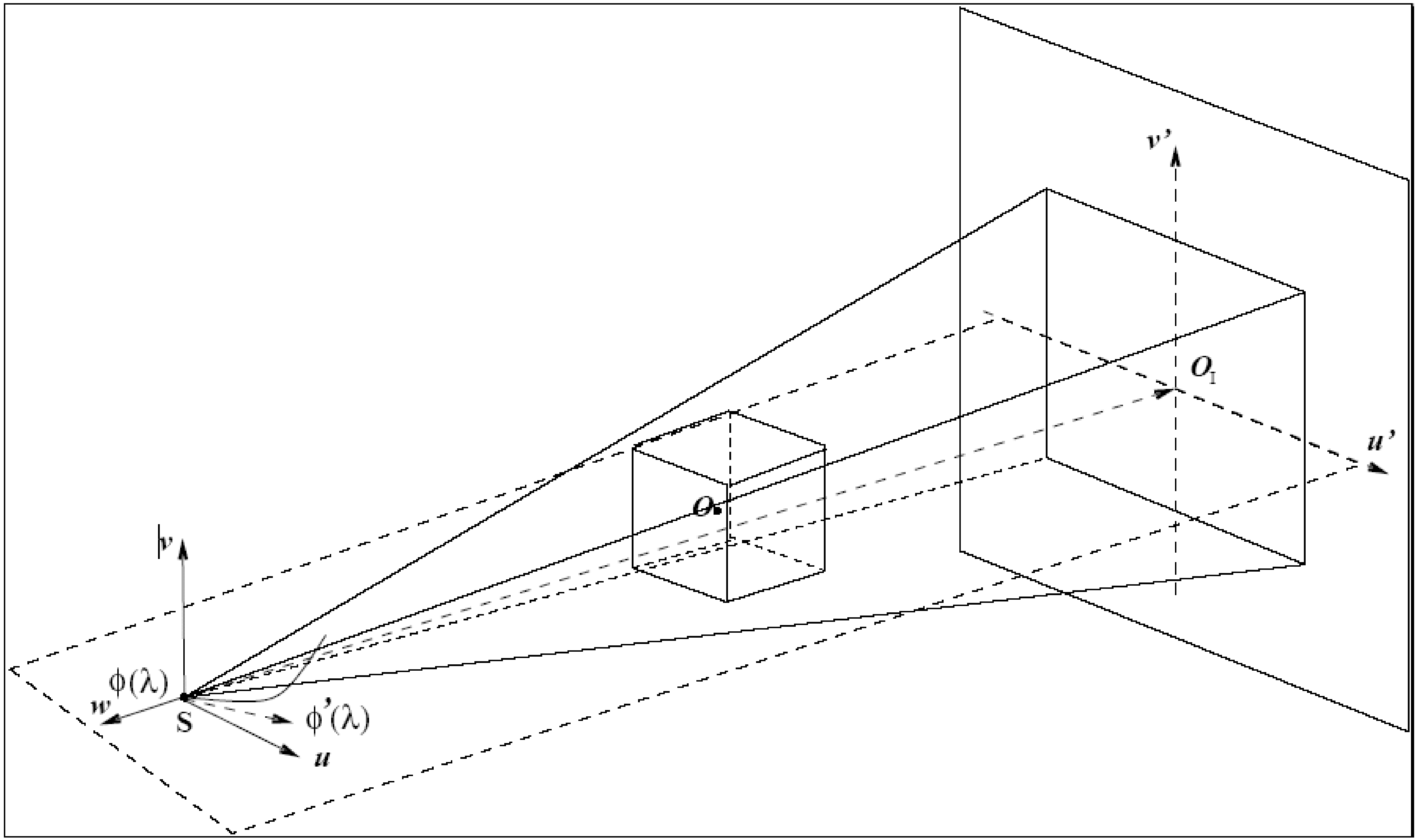
- Parallel Beam → Fan Beam
- Planar Fan → Cone Beam

Parallel Beam to Fan Beam



Coordinate Transform in 2D Radon Space

Cone Beam Geometry — 3D



Radon's Formula

- In 2D: ~ derivatives of line integrals
- In 3D: derivatives of plane integrals
- Can't get plane integrals from projections

$$\int \left(\int f(r, \theta) dr \right) d\theta$$

$$\int \int \frac{1}{r} f(x, y) \, dx \, dy$$

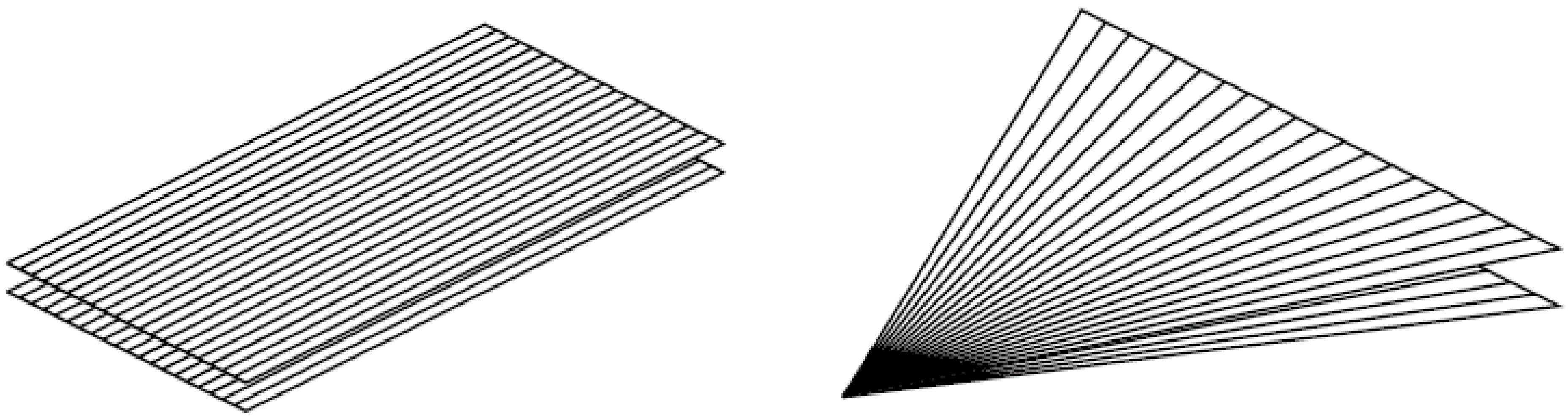
Radon's Formula in 3D

$$f(\boldsymbol{x}) = -\frac{1}{8\pi^2}\int_{S^2} \left.\frac{\partial^2 Rf(l,\boldsymbol{\beta})}{\partial l^2}\right|_{l=\boldsymbol{x}\cdot\boldsymbol{\beta}} d\boldsymbol{\beta}$$

where

$$Rf(l,\boldsymbol{\beta}) = \int f(\boldsymbol{x})\,\delta(\boldsymbol{x}\cdot\boldsymbol{\beta} - l)\,dV$$

Grangeat's Trick

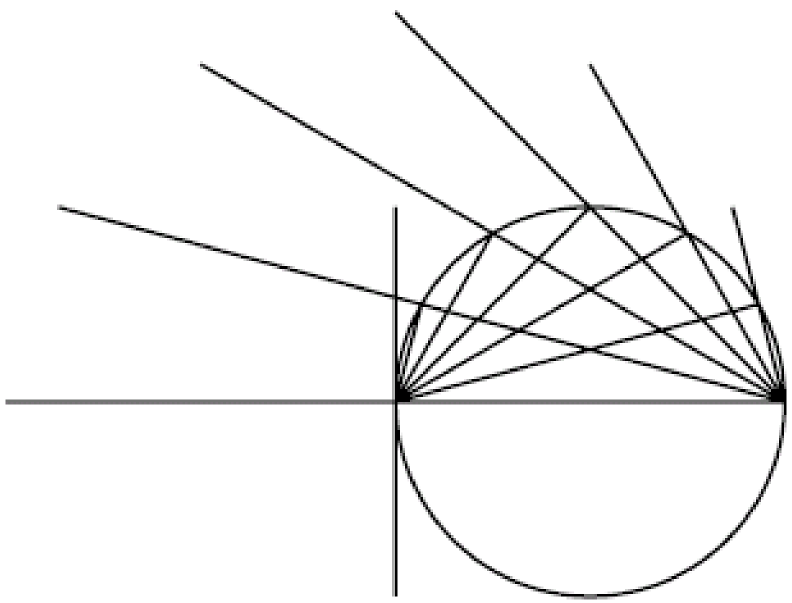


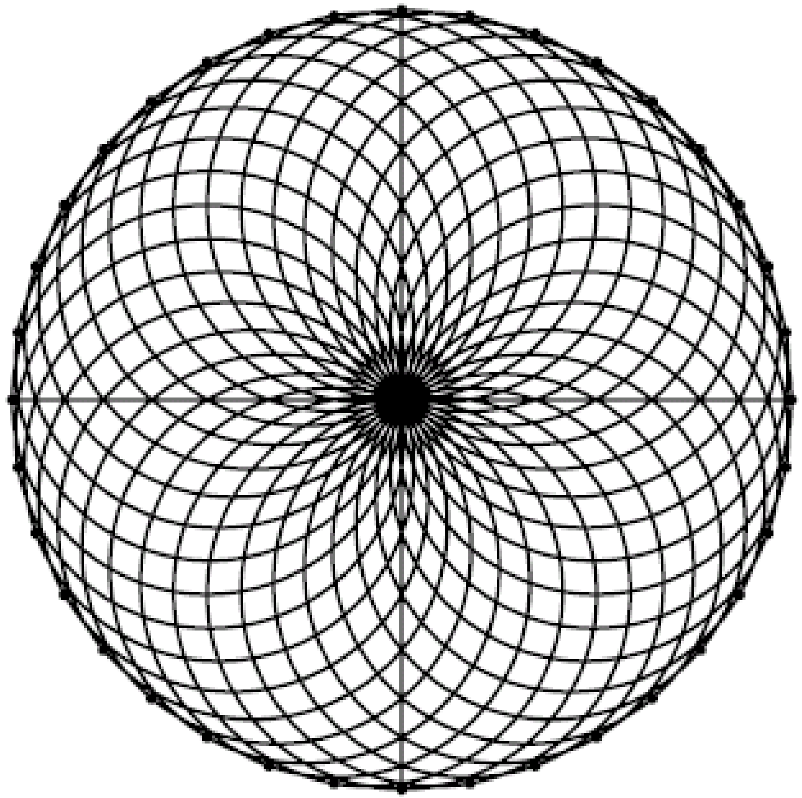
$$\frac{\partial}{\partial z}\int\int f(x,y,z)\,dx\,dy = \frac{\partial}{\partial\theta}\int\int f(r,\phi,\theta)\,dr\,d\phi$$

Exact Cone Beam Reconstruction

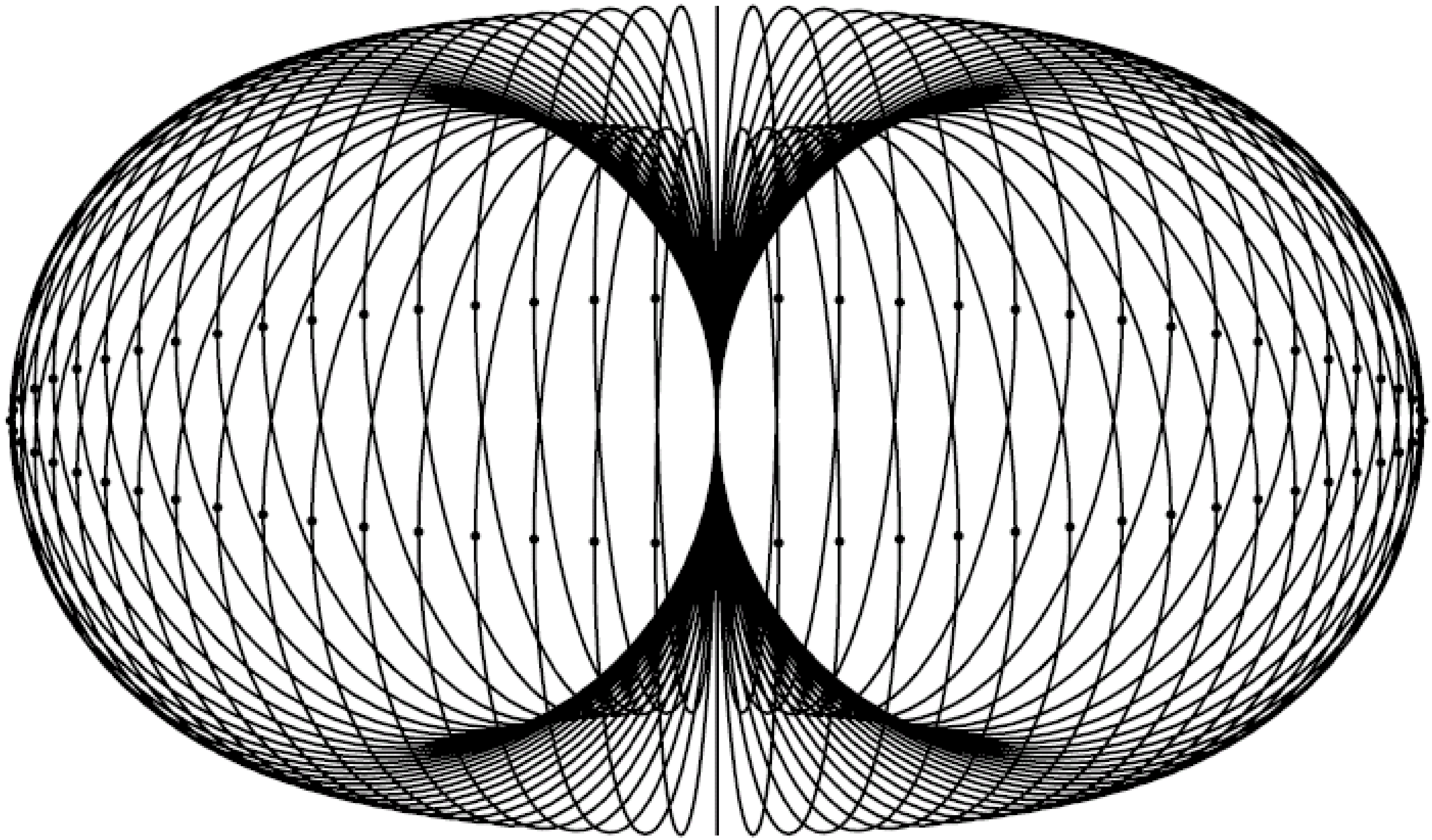
- Data Sufficiency Condition
- Good “Orbit” for Radiation Source

Radon Space — 2D

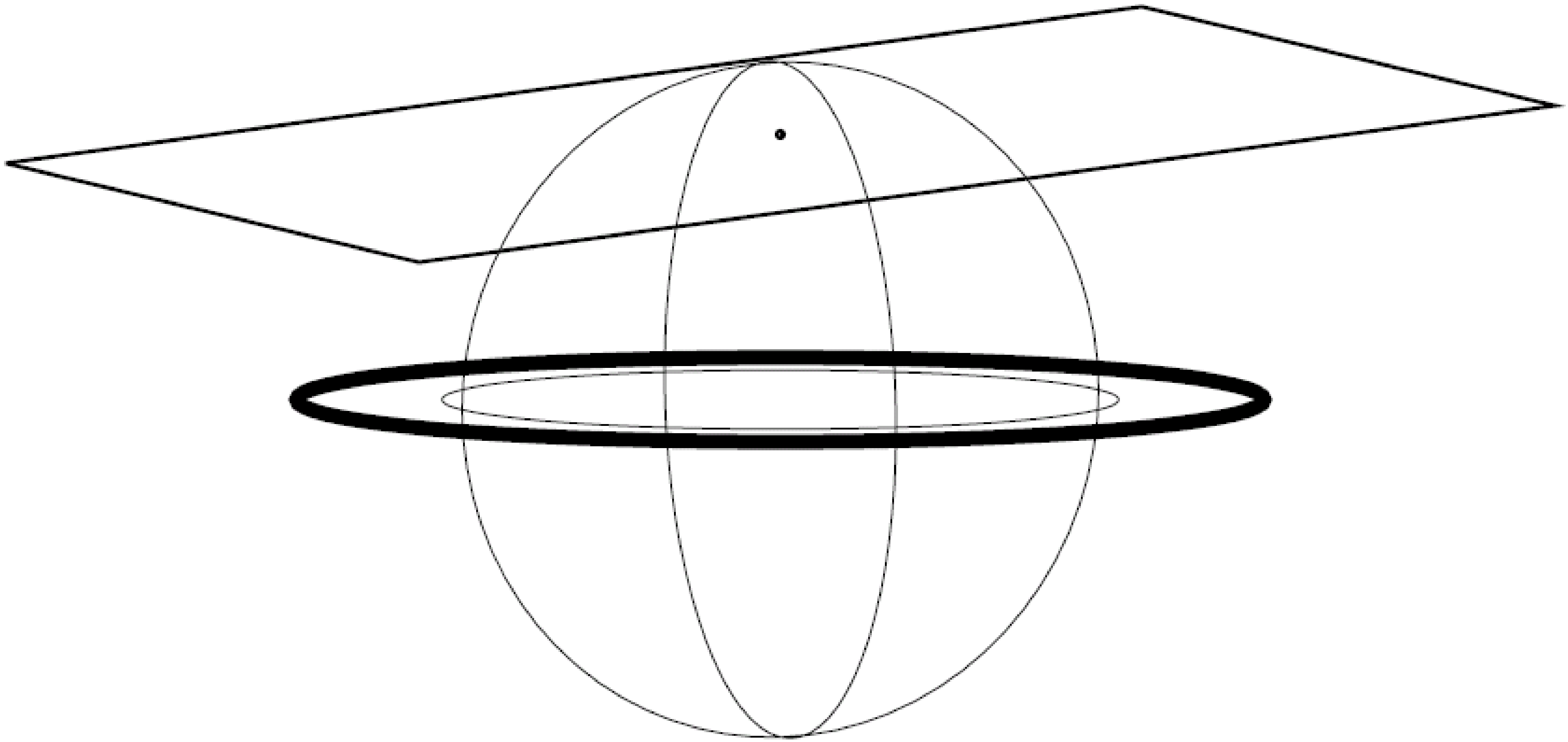




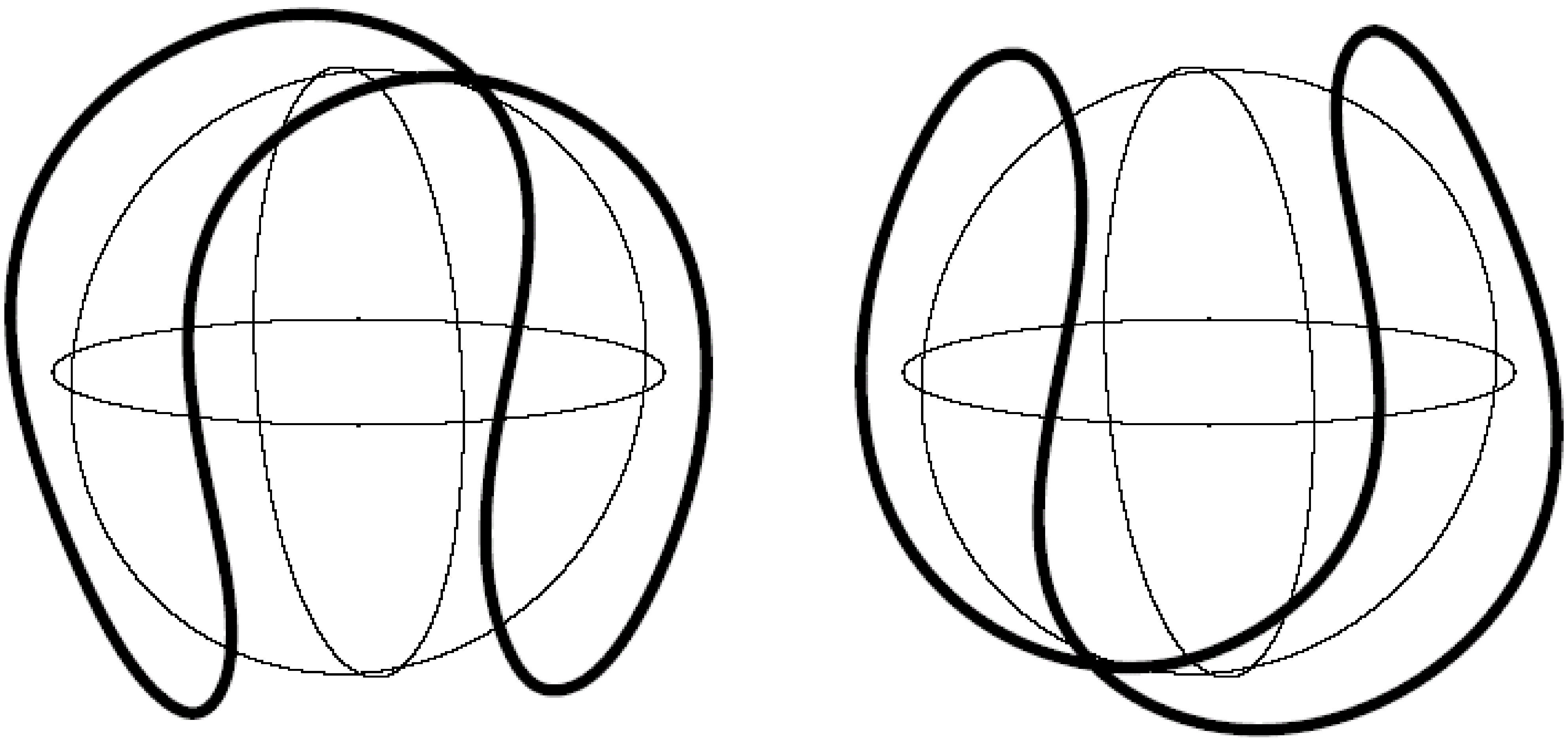
Circular Orbit is Inadequate (3D)



Data Insufficiency



Good Source Orbit



Exact Cone Beam Reconstruction

- Data Sufficiency Condition
- Good "Orbit" for Radiation Source

- Practical Issue: Spiral CT Scanners
- Practical Issue: "Long Body" Problem

COMPUTATIONAL IMAGING

(1) Synthetic Aperture Imaging

(2) Coded Aperture Imaging

(3) Diaphanography—Diffuse Tomography

(4) Exact Cone Beam Reconstruction

COMPUTATIONAL IMAGING