MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF COMPUTER SCIENCE AND ELECTRICAL ENGINEERING

6.881 COMPUTATIONAL IMAGING

HWP 2 **Handed out: 2006 Mar 9th** **Due: 2006 Mar 16th**

Name: **email:**

Problem 1: Suppose a synthetic aperture microscope (SAM) has N beams which impinge with the same angle of incidence θ on the reference plane and are equally spaced around a ring. Let the wavelength of the monochromatic coherent light be λ. Show that the highest spatial frequency term in the interference pattern will be

$$\omega_{\text{high}} = \frac{4\pi}{\lambda}\cos\theta$$

if N is even.

What is the spacing between peaks in this component of the interference pattern? What is the ratio of this spacing to the wavelength λ of the monochromatic light? What is the highest frequency term when N is odd?

Show that (aside from the zero frequency or "D.C." term) the lowest spatial frequency in the interference pattern will be

$$\omega_{\text{low}} = \frac{4\pi}{\lambda}\cos\theta\sin\frac{\pi}{N}$$

What is the ratio of the wavelength λ_{low} corresponding to this frequency and the wavelength λ of the monochromatic light? Given this result, comment on the properties of an computed image derived from data collected by such a device.

Problem 2: The frequency $\omega(\theta)$ of the interference pattern between two beams varies with the angle θ between the beams as described in the previous problem. Here we consider the variation in sampling of the transform domain in the limit when there are very many beams.

Express the density of sampling of the frequency domain at radial frequency ω as a function of the angle θ. Note that density equals number of samples per unit area, and that the area allocated to a sample is proportional to $rdr/d\theta$.

Express the density of sampling as a function of radial frequency ω instead of the angle θ.

Clearly the sampling density is high near the lowest and near the highest frequencies. Why do we still consider sampling to be less advantageous in those areas when compared to the mid frequency range?

Problem 3: In an acousto-optical modulator (AOM), a light beam of wavelength λ is incident on a traveling grating of spacing d with incident and emergent angle θ of the first order refraction with respect to the grating lines. Consider the waves to be travelling in the AOM in the vertical direction and the corresponding grating lines to lie in the horizontal plane.

For constructive first order interference of the emergent beam, diffractions from adjacent elements of the diffraction grating must differ by one wavelength. Show that

$$\sin\theta = \frac{\lambda}{2d}$$

Because the grating is moving with velocity of sound v in the material of the AOM, the beam path, and hence the phase shift, is constantly increasing. What is the rate of change of phase resulting from the movement of the grating?

What is the equivalent shift in the frequency of light resulting from this? How is this change in frequency related to the frequency of sound in the acousto-optical modulator?

Note that spatial frequency is given by $(2\pi)/\lambda$, while frequency in the time domain case is given by $(2\pi)/T$, where T is a period. The period of oscillation at a point is related to the wavelength by the velocity of the wave passing that point.

Problem 4: Suppose you wish to produce two spots of light at

$$(x, y) = (\pm L/2, 0)$$

in a synthetic aperture device by simply adding beam complex amplitudes computed for producing the two isolated spots separately. Suppose there are N equally spaced beams and that the wavelength on the reference plane is $\lambda' = \lambda/\cos\theta$.

First determine what the complex amplitude of the beams should be as a function of the angle ϕ that the beam makes with the x-axis in order to produce a bright spot at $(L/2, 0)$. Repeat for the spot at $(-L/2, 0)$.

Now combine the two results while shifting the phase of the spot at $(+L/2, 0)$ by $+\alpha/2$ and shifting the phase of the spot at $(-L/2, 0)$ by $-\alpha/2$.

Do the beams now interfere destructively at the origin?

(Please see next page for Problem 5)

Problem 5: Flip a fair coin 19 times and record the results. Taking "0" to represent tails and "1" to represent heads, construct a binary vector of length 19. Now produce a periodic binary pattern by laying copies of the vector so constructed next to one another. Find the correlation of this periodic pattern with the original vector for all shifts.

Now instead use the quadratic residues for $n = 19$ to define the "1"s in a binary array of length 19. Again, produce a peridoc pattern from this vector and find the correlation of this periodic pattern with the original vector for all shifts.

Comment on the differences between the two results, and the advantages or disadvantages of one or the other in one-dimensional "coded aperture" imaging.

Consider now a one dimensional "random binary mask" of length N where the probability that a particular cell has value "1" is p — and correspondingly the probability that it has value "0" is $(1 - p)$. The probabilities of having a "1" at different positions are independent.

Use this binary vector of length N to create a periodic pattern as above. We'll now correlate the perdiodic pattern so created with the original vector of length N. What is the expected value of the correlation for zero shift?

What is the expected value of the correlation with shift by a number of cells that is not an integer multiple of N?